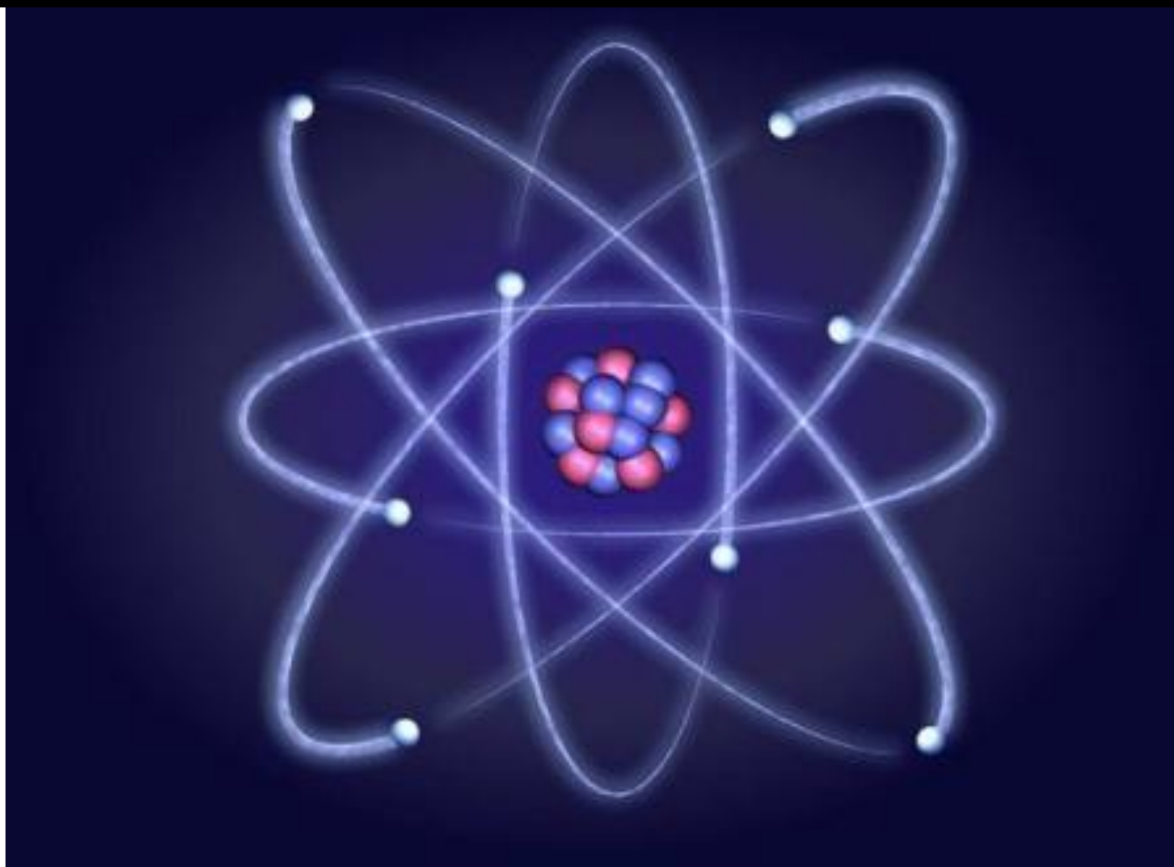
Mid Term Syllabus

Modern Physics



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Mid Term Syllabus

Nuclear Physics

Q. 1.1. (a) Qualitatively explain the important constituents and facts which lead to the development of structure of nucleus.

(b) Discuss electron-proton theory of nuclear composition. Why was it thought? Give at least five causes for the failure of proton electron hypothesis.

Ans. (a) **Structure of the nucleus.** An atom is electrically neutral under normal conditions. But some atoms are radioactive which emit negatively or positively charged particles (β-rays and α-rays). Hence it was assumed that the atoms are made up of equal number of positive and negative charges. In the year 1897. Sir J.J. Thomson discovered the existence of a very small particle which carried a negative charge. This particle was called the **electron**.

Thomson model. Thomson, therefore, assumed that an atom consisted of a sphere of positive charge of uniform density in which an equal and opposite negative charge was symmetrically distributed in the form of electrons.

Rutherford model. The experiments on the scattering of α-particles showed that some of the α-particles were deflected through more than 90° on their passage through a thin gold foil. Rutherford interpreted the observation as an indication of the presence of a very small entity having a strong positive charge on it, which exerts a strong force of repulsion on the α-particle. This small entity. carrying a positive charge was called the **nucleus**. Calculations from experimental results showed that the approach of an α-particle to the nucleus of an atom was of the order of 10^{-14} m. whereas from a study of kinetic theory of gases and other phenomena, the diameter of the atom was shown to be of the order 10^{-10} m.

Rutherford, therefore, came to the conclusion that an atom only consists of a small massive nucleus. which carries a positive charge with a comparatively vast region around containing the electrons. The electrostatic force of attraction of the massive positively charged nucleus was supposed to be balanced by the centrifugal force due to the rotation of the electrons around the nucleus. Thus, Rutherford transformed the static model due to Thomson into a dynamic model in which the nucleus plays the role of the sun and the electrons that of the orbiting planets in the solar system.

(b) Proton-electron-model. Until the discovery of the neutron it was assumed that an atom only contains electrons and protons. The concept of proton-election constitution of the nucleus arose from the following experimentally observed facts:

(i) Some radioactive elements emitted β-rays which were found to be fast moving **electrons**.

(ii) The discovery of isotopes and their whole number atomic weights suggested that all elements wcrc built out of hydrogen atoms. The fractional atomic weights are due to the presence of two or more isotopes each of which has an integral mass. The experimental results show that nuclei of different elements can be regarded as being a collection of an integral number of hydrogen nuclei.

The nucleus of hydrogen has a positive charge equal in magnitude to that of the electron and is known as a **proton**.

According to *Proton-electron model*, a nucleus of atomic mass number A and atomic charge number Z consisted of A protons and (A - Z) electrons, thus giving it a net charge +Ze. The nucleus was supposed to be surrounded by Z extra-nuclear electrons. thereby making the atom on the whole neutral. For example. an atom of nitrogen of atomic weight 14 and atomic number 7 was supposed to contain 14 protons and 14 electrons. 14 protons and 7 electrons are packed together in the small space of the nucleus and the remaining seven electrons revolve round the nucleus in suitable orbits. **Failure of the electron-proton hypothesis.** Although proton-electron model had some satisfactory aspects, yet them were a number of objections against the model. The most serious objection is that the electron cannot exist within the nucleus.
The reasons are discussed in Q. 1.25.

Q. 1.2. (a) What is proton-neutron hypothesis? Give reasons for acceptance of this hypothesis for the constitution of the nucleus. How does this hypothesis avoid the failures of proton-electron hypothesis?

(b) Give any two evidences of neutron proton model of the nucleus.

Ans. (a) Until the discovery of neutron it was assumed that a nucleus only contains electrons and protons. This set up of the nucleus has changed with the discovery of the neutron.

Neutron. In 1930. Bothe and Becker bombarded Beryllium with α-particles and found that a very penetrating radiation was produced. J. Chadwick. in 1932 showed that this radiation consisted of particles of mass slightly greater than that of the proton but carried no charge because its path was not deflected by electric and magnetic fields. The particle also did not ionize a gas. This new fundamental particle was given the name **neutron**.

Proton-neutron hypothesis. With the discovery of the neutron the set-up of the nucleus has changed. To overcome the difficulties experienced by proton-electron model. Heisenberg proposed a new hypothesis that the nucleus consists of protons and neutrons. According to proton-neutron hypothesis, a nucleus of atomic mass (number) A and atomic charge (number) Z consisted of protons giving it a net charge + Ze and N = (A - Z) neutrons giving it a total mass (N + Z)= A. For example, the nitrogen nucleus contains 7 protons and 7 neutrons, thus making atomic weight 14 and atomic number 7. The number of extra nuclear electrons is 7 i.e. = Z, thus making the atom, on the whole, neutral. As the nucleus of an atom contains protons and neutrons a common name has been given to these particles, i.e. **nucleons**. In $_{92}U^{238}$ nucleus there are in all 238 nucleons out of which 92 are protons and 238 - 92 = 146 are neutrons. The rest mass of the proton is equal to 1.0072766 a.m.u. = 1.6725×10^{-27} kg and that of the neutron is equal to 1.0086654 a.m.u. = 1.6748×10^{-27} kg. The proton and neutron are, therefore. two charge states of the same particle - the

nucleon. The proton is the protonic state with charge + e and the neutron is the neutronic state with charge zero.
The proton-neutron theory has overcome the objections of the proton-electron theory thereby avoiding the failures of proton-electron hypothesis and paving the way for acceptance of proton neutron hypothesis.

Avoiding failures of Proton-Electron Hypothesis

(i) Isotopic mass. The isotopic masses can be easily explained on the basis of proton-neutron hypothesis. Isotopes of the same atom have the same Z i.e., the same number of protons but different atomic masses i.e., A. Thus they have only a different number of neutrons i.e., N= (A - Z).

(ii) Nuclear spin. It has resolved the difficulty regarding nuclear spin. Proton and neutron are both Fermions and have a spin $\frac{1}{2}\hbar$. If the nucleus contains an even number of nucleons (protons + neutrons) i.e., A is even like N_7^{14}, O_8^{16} it has an integral or zero spin. On the other hand, if the nucleus contains an odd number of nucleons like $_4Be^9$ having nine (odd) particles it has a half-integral spin. This agrees with experimental values and explains the observed spin of deutron equal to one.

(iii) Nuclear magnetic dipole moment. The magnetic dipole moment associated with nuclear spin $= \frac{e\hbar}{2m_p}$ where m_p is the rest mass of the proton. The quantity $\frac{e\hbar}{2m_p}$ is called nuclear magneton and is denoted as β_n. It has a value 3.15×10^{-8} eV/T (or 5.05×10^{-27} J/T). The magnetic dipole moment associated with a spinning electron is $\frac{e\hbar}{2m_e}$ where m_e is the rest mass of the electron.
The quantity $\frac{e\hbar}{2m_e}$ is called Bohr magneton μ_B and has a value 5.79×10^{-5} eV/T (or 9.27×10^{-24} J/T). The ratio of a nuclear magneton to Bohr magneton is $\frac{m_e}{m_p} = \frac{1}{1836}$ i.e., a nuclear magneton is almost 2000 times smaller than Bohr magneton.
A neutron has a magnetic moment equal to -1.913 β_n. As it carries no charge it cannot posses any magnetic moment due to orbital motion. A proton has an intrinsic magnetic moment 2.792 β_n. Its magnetic moment due to orbital motion is of the order of β_n. Thus the resultant magnetic moment of the nucleus should be of the order of nuclear magneton.
The observed magnetic moment of the nucleus is also of the order of nuclear magneton lying between $-2\beta_n$ to $+ 4\beta_n$. Thus the proton-neutron hypothesis avoids the difficulty of explaining the nuclear magnetic moment on the basis of proton-electron theory.

Acceptance of proton-neutron theory. (i) This theory removes the difficulty of having an electron in the nucleus and is based upon two actually existing fundamental particles, the proton and the neutron.
(ii) The presence of protons and neutrons in the nucleus is in accordance with Heisenberg's uncertainty principle, according to which

$$\Delta x.\Delta p = \frac{\hbar}{2}$$

where Δp is the uncertainty in momentum. As Δx is the nuclear radius ≥ 10^{-14} m, the value of

$$\Delta p = \frac{\hbar}{2\Delta x} = \frac{1.055 \times 10^{-34}}{2 \times 10^{-14}} = 5.3 \times 10^{-21}\, kgms^{-1}$$

The rest mass of the proton (or neutron) is of the order of $m_p = 1.67 \times 10^{-27}$ kg. Hence the corresponding value of velocity $v = \frac{5.3 \times 10^{-21}}{1.67 \times 10^{-27}} = 3 \times 10^5\, ms^{-1}$. It is, therefore, a non-relativistic case. The value of kinetic energy is given by

$$E_k = \frac{P^2}{2m} = \frac{(5.3 \times 10^{-21})^2}{2 \times 1.67 \times 10^{-27}} = 8.4 \times 10^{-15}\, Joule = 52\, KeV$$

Since the energy carried by the protons or neutrons emitted by the nuclei is greater than 52 KeV the protons and neutrons can exist within the nucleus.

(iii) The process of β-emission i.e., emission of electrons from the nuclei is explained on the basis of conversion of a neutron into a proton as these are two charge states of the same particle-the nucleon. An electron does not pre-exist in a nucleus but is formed just at the instant of emission, by the transformation of a neutron into a proton according to the equation.

$$n \rightarrow p + e^-$$

The positron emission is also explained on the same basis by the conversion of proton into a neutron according to the relation.

$$p \rightarrow n + e^+$$

(iv) Classification of elements. This theory also leads to more refined classifications of elements as follows:

(a) **Isotopes.** These are nuclei having the same atomic (proton) number Z but different atomic masses (A) e.g., $_8O^{16}$ and $_8O^{17}$. The isotopes of an element contain the same number of protons and different number of neutrons.
All the isotopes of an element have identical chemical properties and differ physically only in mass.

(b) **Isotones.** These are nuclei having the same neutron number N e.g., $_6C^{13}$ and $_7N^{14}$. Both have (13 - 6) = 7 or (14 – 7) = 7 neutrons.

(c) **Isobars.** These are nuclei having the same mass number A (total number of nucleons i.e., proton +neutrons) but different atomic number Z, e.g., $_6C^{14}$ and $_7N^{14}$. Both have the same mass number A = 14 but Z = 6 for $_6C^{14}$ and Z = 7 for $_7N^{14}$.

(d) **Isomers.** These are atoms which have the same atomic mass A and same atomic number Z but differ from one another in their nuclear energy states. They also exhibit differences in their internal structure and have different life times. They are also known as isomeric nuclei.

(e) **Mirror nuclei.** Nuclei with same mass number A but with proton and neutron number interchanged i.e., the number of protons in one is equal to the number of neutrons in the other, are called mirror nuclei e.g., $_4Be^7$ (Z = 4, N = 3) and $_3Li^7$ (Z = 3, N = 4).

Q. 1.3. (a) Write a short note on Isotopes.

(b) A nucleus emits an α-particle followed by two β-particles. Show that the final nucleus is an isotope of the original one.

Ans. (a) Isotopes. Isotopes are nuclei having the same atomic (proton) number Z but different atomic mass A e.g., $_8O^{16}$ and $_8O^{17}$. The isotopes of an element contain the same number of protons but different number of neutrons.

Hydrogen has three isotopes, (i) Hydrogen $^1H_1$; (ii) Deuterium $^2H_1$ and (iii) Tritium $^3H_1$. The nucleus of each isotope of hydrogen contains one proton, but a deuterium nucleus ($^2H_1$) contains a proton as well as a neutron in tritium nucleus ($^3H_1$) contains one proton and two neutrons.

At present about 297 different isotopes, from hydrogen (mass no. 1) to uranium (mass no. 238) including radioactive ones are known. About 300 unstable isotopes have been produced by disintegration. Some elements like As, F, I and Au have only a single isotope where as other like Xenon and Hg have nine each. The atomic weights of the isotopes of Xenon are 124, 126, 128, 129, 130, 131, 132, 134 and 136. The relative abundance of various isotopes is found from the intensity of lines obtained from Aston's mass spectrograph. In general, elements of odd atomic numbers do not have more than 2 stable isotopes whereas those of even atomic numbers usually have a much larger number.

Most of the physical and chemical properties of an element are determined by the number and arrangement of the electrons in its atoms. Since the isotopes of an element have almost identical electron structures, the two isotopes of chlorine, for instance, have the same yellow colour, the same suffocating odour, the same bleaching effect, and the same ability to combine chemically with metals. Because boiling and freezing points depend somewhat on atomic mass, these properties differ slightly between the isotopes, as does density.

(b) When a nucleus emits an α-particle and then two β-particles, the reaction can be represented as

$$_ZX^A = {}^{A-4}X_{Z-2} + {}_2He^4 \ (\alpha\text{-particle})$$
$$= {}_zX^{A-4} + 2e_{-1}^0 \quad (2 \text{ electrons})$$

Now $_ZX^A$ and $_zX^{A-4}$ have the same atomic number Z i.e., they have the same number of protons and differ only in their neutron number. Thus the final nucleus is an isotope of the original nucleus when it emits an α-particle and then two electrons.

Q. 1.4. (a) Give the properties of the nucleus. Explain the terms: Angular momentum of the nucleus, nuclear magnetic dipole moment defining a nuclear magneton and nuclear electrical quadrupole moment.

(b) Explain the wave mechanical properties of the nucleus

(i) Statistics and (ii) Parity.

Ans. (a) Properties of nuclei. (1) Charge. If Z is the charge number of nucleus i.e., the number of protons in it, then

Charge on the nucleus = + Ze

where e is the positive charge equal to the charge on the electron = 1.6×10^{-19} Coulomb.

(ii) Mass. If A is the mass number i.e., the total number of nucleons in the nucleus i.e., Z protons and (A - Z) neutrons, then the mass of the nucleus is very nearly equal to A atomic mass units. In terms of atomic units, the mass of carbon atom C^{12} is taken to be = 12 (a.m.u.) and 1 a.m.u. (also written as 1 u) = 1.6604×10^{-27} kg = 931.48 MeV.

∴ Nuclear mass $M_N = A\, m_N$ where m_N is the mass of a nucleon

(iii) Radius. As the nucleus is approximately spherical, its volume is proportional to the total number of nucleons in it or its mass number A.

$$\frac{4}{3}\pi r^3 \propto A$$

where r is the radius of the nucleus.

Hence $$r \propto A^{1/3}$$

or $$r = r_o A^{1/3}$$

where $r_o = 1.3 \times 10^{-15}$ m = 1.3 fermi [1 fermi (fm) = 10^{-15} m]

(iv) Density. Nuclear density $\rho_N = \frac{Nuclear\ mass}{Nuclear\ volume}$

Nuclear mass = Am_N where A = mass number and m_N = mass of the nucleon

= 1.67×10^{-27} kg.

$$\text{Nuclear volume} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (r_o A^{1/3})^3 = \frac{4}{3}\pi {r_o}^3 A$$

$$\rho_N = \frac{Am_N}{\frac{4}{3}\pi {r_o}^3 A} = \frac{m_N}{\frac{4}{3}\pi {r_o}^3} = \frac{1.67\ \times 10^{-27}}{\frac{4}{3}\pi (1.3\ \times 10^{-15})^3}$$

$$= 1.816\ \times 10^{17}\ kgm^{-3}$$

This shows that the nuclear matter is in a highly compressed state.

As the density of the nucleus is independent of A, its value is almost the same for all nuclei.

In other words, nuclear mass is proportional to nuclear volume or size.

Question. Assuming 1 a.m.u. = 1.67 × 10^{-27} kg. estimate the density of nuclear matter.

(v) Binding energy. The nucleons exert short range nuclear forces of attraction on each other. When any two particles attract each other, the sum of their masses, when separated, exceeds that of the bound system since energy (or mass) must be added to the system to separate it into component particles.

This energy is called binding energy and holds the nucleons together to form a stable nucleus. Thus the stability of the nucleus is due to the decrease in mass of the constituent particles which combine to form the nucleus.

For a nucleus having Z protons and N= (A — Z) neutrons;

Assumed nuclear mass = $Zm_p + Nm_n$

where m_p is the mass of the proton and m_n that of the neutron. The real nuclear mass M_N is, however, less than the assumed nuclear mass. The difference in assumed and real mass $(Zm_p + Nm_n) - M_N = \Delta m$ gives the decrease in mass and the binding energy is given by

Binding energy B.E = $\Delta mc^2 = c^2\,[(Zm_p + Nm_n) - M_N]$

(vi) Angular momentum of the nucleons. 1. Spin angular momentum. Each nucleon i.e., each proton and each neutron in the nucleus has an intrinsic angular momentum which may be pictured as being caused by particle's spinning motion about an axis through its centre of mass. This is known as spin angular momentum and its magnitude is given by

$$|\vec{s}| = \sqrt{s\,(s+1)}\hbar$$

where s = 1/2 is the spin angular momentum quantum number.

The magnetic spin quantum number m_s is the component of s in a specified quantization direction-that of the applied magnetic field usually taken as Z-axis. With s $=\frac{1}{2}$, m_s = $+\frac{1}{2}$ and $-\frac{1}{2}$ corresponding to parallel and anti-parallel alignment with respect to Z-axis.

2. Orbital angular momentum. In addition, each individual nucleon may be pictured as having an angular momentum associated with its orbital motion within the nucleus. This is known as orbital angular momentum and its magnitude is given by

$$|\vec{l}| = \sqrt{l\,(l+1)}\hbar$$

where l is the orbital angular momentum quantum number having only integral values 0,1,2,3, ...

The orbital magnetic quantum number m_l is the component of l in the specified quantisation direction - that of applied magnetic field unusaually taken as Z-axis. m_l can take $(2l + 1)$ possible values such as, l, $(l$ - 1) + 1, 0, - 1, - $(l$ - 1), - l.

3. Total angular momentum. The total angular momentum $\vec{j}$ of a nucleon is the vector sum of its orbital and spin angular momenta

$$\therefore\ \vec{l} = \vec{l} + \vec{s}$$

The magnitude of the total angular momentum of a nucleon is given by

$$|\vec{j}| = \sqrt{j\,(j+1)}\hbar$$

where j is the total angular momentum quantum number. For both proton and neutron which are Fermions s = 1/2 and l is an integer. Hence j can take on the values 1/2, 3/2, 5/2, i.e., j is always half integral.

The magnetic total angular momentum quantum number m_j is the component of j in the specified quantization direction — that of the applied magnetic field usually taken as the Z-axis. m_j can take (2j + 1) possible values such as j, (j - 1) … - (j - 1), -j.

4. Total angular momentum of the nucleus (nuclear spin). The total angular momentum of the nucleus $\vec{I}$ for a particular nuclear state is the resultant of the individual total angular momenta of all the constituent nucleons in the nucleus. The magnitude of the total angular momentum of the nucleus is given by

$$|\vec{I}| = \sqrt{I\,(I+1)}\hbar$$

where I is the total angular momentum quantum number for the nucleus. The value of I depends upon the type of interaction or coupling between the nucleons.

I generally called **nuclear spin** is an integral multiple of $\hbar(h/2\pi)$ if A is even and an odd half-integral multiple of $\hbar$ when A is odd. If both Z and A are even the value of I in the ground state is always zero. All odd-odd nuclei i.e., nuclei with odd number of protons and odd number of neutrons and hence even A have an integral spin I. On the other hand, all odd-even nuclei i.e., nuclei with odd number of protons and even number of neutrons or odd number of neutrons and even number of protons and hence odd A have half integral spin lying between $\hbar/2$ and 9 $\hbar/2$.

(vii) Magnetic moment. The magnetic dipole moment associated with nuclear spin = $\frac{e\hbar}{2m_p}$ where m_p is the rest mass of the proton. The quantity $\frac{e\hbar}{2m_p}$ is called nuclear magneton and is denoted as β_n.

$$\beta_n = \frac{e\hbar}{2m_p} = \frac{1.6 \times 10^{-19}C \times 6.58 \times 10^{-16}eV}{2 \times 1.673 \times 10^{-27}kg}$$
$$= 3.15 \times 10^{-8}\ eV/T$$
$$= 5.5 \times 10^{-27}\ J/T$$

The magnetic dipole moment associated with a spinning electron is $\frac{e\hbar}{2m_e}$ where m_e is the rest mass of the electron. The quantity $\frac{e\hbar}{2m_e}$ is called Bohr magneton μ_B and has a value

$$= 5.79 \times 10^{-5}\ {}^{eV}/_{T}\ (or\ 9.27 \times 10^{-27}J/T)$$

The ratio of a nuclear magneton to a Bhor magneton

$$\frac{\beta_n}{\mu_B} = \frac{m_e}{m_p} = \frac{1}{1836}$$

For all nuclei magnetic moment μ_N is given by

$$\mu_N = g\frac{e\hbar}{2m_p}\frac{\vec{I}}{\hbar}$$

Where $\vec{I}$ is the total angular momentum of the nucleus.

The g-factor varies from nucleus to nucleus. The magnetic dipole moment of a proton is given by

$$\mu_{proton} = 2.792\frac{e\hbar}{2m_p} = 2.792\beta_n$$

The magnetic dipole moment of a neutron is given by

$$\mu_{neutron} = -1.913\frac{e\hbar}{2m_p} = -1.913\beta_n$$

Having a positive charge, the magnetic field around a proton is parallel to the direction of its total angular momentum and hence its magnetic moment is positive. A negative magnetic moment of a neutron is a clear indication that the neutron is a complex particle containing negative and positive charges in equal amounts and that the negative charge is, on the average farther from the axis of rotation.

As an example, consider the case of a deutron. It is a nuclear particle composed of one proton and one neutron. Its known spin I = 1 i.e., the spins of the two nucleons are parallel. Since the two magnetic moments are oppositely directed the resultant magnetic moment should be 2.792 - 1.913 = 0.879 β_n. Precision measurements show that μ_d = 0.857β_n.

The difference is well outside the experimental error and has been explained by assuming that the ground state of deutron is not a simple $l = 0$ state.
The magnetic moment of *odd-even* and *even-odd* nuclei is due to the unpaired single nucleon. In general, the magnetic moment of the nucleus is about one thousandth of the magnetic moment of the electron
(viii) Electric quadruple moment. It is found that nuclei do not have electric dipole moment but electric quadruple moments have been observed in a number of nuclei. The electric quadruple moment of a nuclear charge distribution which is symmetric about the Z-axis is given by

$$Q = \frac{1}{e}\int_V (3z^2 - r^2)\,\rho(x,y,z)dv$$

where $\rho(x,y,z)$ is the charge density and $r^2 = x^2 + y^2 + z^2$. For uniformly charged ellipsoid of revolution defined by the equation

$$\frac{x^2 + y^2}{a^2} + \frac{z^2}{b^2} = 1$$

the electric quadruple moment reduces to

$$Q = \frac{2}{5}Z(b^2 - a^2)$$

where Ze is the total nuclear charge.

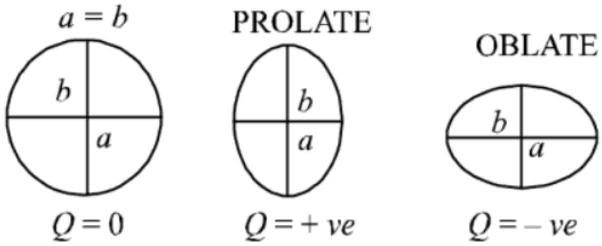


Figure 1

Evidently Q = 0 for a spherically symmetric charge distribution (a = b). For a charge distribution stretched in the Z-direction (spin axis) (b > a) i.e., prolate spheroid Q is positive and Q is negative for an oblate spheroid (b < a) i.e., nucleus flattened along the spin axis Dimension of the quadruple moment is that of area. In nuclear physics area is measured in barn. (1 barn = 10^{-28} m^2). Quadruple moment is also measured in fm^2 and its value mostly lies in the range 10^{-28} m^2 for ^{123}Sb to 8×10^{-28} m^2 for ^{176}Lu while deutron has value $Q = 2.74 \times 10^{-31}$ m^2.
(b) Wave mechanical properties. The nucleus has two wave mechanical properties (i) Statistics and (ii) Parity.
Statistics. The quantum mechanical description of a system with a number of particles like the nucleus is given by either Bose-Einstein or fermi-Dirac statistics.
Bose-Einstein Statistics. All particles with integral spin (in units of $\hbar$) or zero obey Bose-Einstein (B.E) Statistics and all called bosons e.g., photon, π meson and deutron. All nuclei with even mass number A obey B.E. statistics. The wave function of a system obeying Bose-Einstein Statistics is symmetric. This means wave function for such a system remains unaltered by the interchange of

all the co-ordinates for any pair of identical particles. Thus, if Ψ (x_1 ..., x_i ..., x_j ... x_n) is the wave function of a system of n identical particles obeying Bose-Einstein Statistics and x_i stands for all the co-ordinates of the particles *i*, the new wave function resulting from interchange of *i* and *j* will be given by Ψ (x_1 ..., x_j ..., x_i ... x_n). In other words

$$\Psi (x_1 \ldots, x_i \ldots, x_j \ldots x_n) = \Psi (x_1 \ldots, x_j \ldots, x_i \ldots x_n)$$

Fermi-Dirac statistics. All particles with half integral spin (ħ/2) obey Fermi-Dirac(F.D.) statistics and are called fermions e.g., electrons, protons and neutrons . All nuclei with odd mass number A obey F.D. Statistics. All fermions obey Pauli's exclusion principle.

The wave function of a system obeying Fermi-Dirac statistics is anti-symmetric. This means that if all the co-ordinates of any pair of identical particles are interchanged in the wave function, the new system will be identical with the original except for a change of sign in the wave function. In other words

$$\Psi (x_1 \ldots, x_i \ldots, x_j \ldots x_n) = -\Psi (x_1 \ldots, x_j \ldots, x_i \ldots x_n)$$

Parity. The parity of a system refers to the behaviour of the wave function Ψ under inversion of co-ordinates through the origin i.e., When x is replaced -*x*, *y* by -*y* and *z* by -*z*.

If the change of sign of *x*, *y*, *z* does not change the sign of the wave function i.e.,

$$\Psi (x, y, z) = \Psi (-x, -y, -z)$$

Then the wave function (or the particle with which it is associated) is to said even or positive parity. If the change of sign of *x*, *y*, *z* changes the sign of the wave function i.e.,

$$\Psi (x, y, z) = -\Psi (-x, -y, -z)$$

Then the wave function (or the particle) is said to have odd or negative parity.

In general, $\Psi (x, y, z) = P\Psi (-x, -y, -z)$

Where p = ± 1. P can be taken as a quantum number and the property defined by it is called the parity of the system.

P = + 1means positive or even parity

P = - 1 means negative or odd parity

In the case of hydrogen like atoms, it is found that parity is related to the orbital quantum number *l* and is given by

$$P = (-1)^l$$

Hence if *l* is even Ψ does not change sign, the parity is positive or even. if *l* is odd Ψ does change sign, the parity is negative or odd. The intrinsic parity of the proton, neutron, neutrino and μ-meson is even whereas the intrinsic parity of *π* meson is odd. The parity of a whole system is the product of the parities of individual parities.

A system having even number of odd parity particles and any number of even parity particles will have even parity. A system with an odd number of odd parity particles and any number of even parity particles will have odd parity.

Parity is a purely quantum mechanical concept having no classical analogue. Nuclear states ae characterized by a defined parity which may be different for different states of the same nucleus. In a nuclear reaction parity remains conserved.

Q. 1.5. Give the significance of (*i*) principal quantum number (*ii*) angular momentum quantum number in relation to individual nucleons.

Ans. (*i*) principal quantum number. The principal quantum number of the individual nucleon is similar to the principal quantum number for the electronic orbit. It characterizes the radial part of the nuclear wave function and is denoted by n. the value of n = 1, 2, 3, … etc.

(b). Angular momentum is described in previous questions.

Q. 1.6. (a) Calculate the mass number of the nucleus whose radius is

(*i*) 4.8×10^{-15}m **(ii) 3.66×10^{-15}m**

given r_o=1.3 fm

(b) Justify the validity of the relation $r = r_oA^{1/3}$

Ans. (a) $r = r_oA^{1/3}$

$r^3 = r_o^3 A$

$$Hence\ A = (\frac{r}{r_o})^3$$

$$(i) \qquad A = (\frac{4.8 \times 10^{-15}}{1.3 \times 10^{-15}})^3 = (\frac{4.8}{1.3})^3 = 50.3 \therefore A = 50$$

$$(ii) \qquad A = (\frac{3.46 \times 10^{-15}}{1.3 \times 10^{-15}})^3 = (\frac{3.66}{1.3})^3 = 22.3 \therefore A = 22$$

(b). Discussed in precious Question.

Q. 1.7. (a) Assuming that the protons and neutrons possess equal masses calculate how many times the nuclear matter is denser than water.

Nuclear radius is = 1.2×10^{-15} $A^{1/3}$ m, where A is the mass number. Mass of proton 1.67×10^{-27} Kg.

Hence show that nuclei of atoms have constant density or nuclear density is independent of mass number.

(b) All matter is nothing but atoms and molecules. Why do we have substances with density very low as compared to nuclear density?

Ans. (a) Nuclear density = $\frac{Nuclear\ mass}{Nuclear\ volume}$

Nuclear mass = Am_N

where m_N = mass of the nucleon = 1.67×10^{-27} kg and A = Atomic mass number.

Nuclear volume = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(r_oA^{1/3})^3 = \frac{4}{3}\pi(1.2 \times 10^{-15})^3 A$ and

$$Nuclear\ density = \rho_N = \frac{Am_N}{\frac{4}{3}\pi r_o{}^3 A} = \frac{m_N}{\frac{4}{3}\pi r_o{}^3} = \frac{1.67 \times 10^{-27}}{\frac{4}{3}\pi(1.2 \times 10^{-15})^3} = 2.3 \times 10^{17}\ kgm^{-3}$$

$$Again \quad \frac{Nuclear\ density}{Density\ of\ water} = \frac{2.3 \times 10^{17}}{1 \times 10^3} = 2.3 \times 10^{14}$$

Hence nuclear matter is 2.3×10^{14} times heavier than water. This shows that nuclear matter is in a highly compressed state.

Exercise. Estimate the nuclear density if $m_n = m_p$ = 1 a.m.u.; R = 1.2×10^{-15} m

Hint. Take 1 u = 1.67×10^{-27} kg. Then nuclear density = 2.3×10^{17} kg m^{-3} as calculated above.

Nuclei of atoms have constant density. The density of nucleus $\rho_N = \frac{m_N}{\frac{4}{3}\pi r_o{}^3}$ and is independent of A the mass number. As the density of nucleus is independent of A the atomic mass, its value is almost the same for all nuclei. In other words, nuclei of atoms have constant density.

(b) Low density of bulk matter. An atom consists of a positively charged nucleus of which the radius is of the order of 10^{-15} m whereas the radius of the atom is of the order of 10^{-10} m. The whole mass of the atom is concentrated at the nucleus, the orbital electrons contributing only a negligible mass. The space between the orbital electrons and the nucleus is perfect vacuum.

The radius of the atom is thus 10^5 times that of the nucleus and its volume 10^{15} times (being proportional to r^3).

Hence average density of matter = $\frac{2.3 \times 10^{17}}{10^{15}}$ = 230 kg/m^3. This density is very low as compared to the nuclear density.

Q. 1.8. (a) Find the density of $_{12}C^6$ nucleus.

(b) Comment on the following properties of $^{208}Pb_{82}$ nucleus (*i*) charge (*ii*) spin (*iii*) size

Ans. (a) The radius of a nucleus is given by R = 1.2 $A^{1/3}$ fm

where fm = Fermi = 10^{-15} metre (It is also called a fermtometer).

∴ Radius of carbon nucleus

$$R = 1.2 \times 10^{-15} \text{ x } 12^{1/3} = 2.7 \text{ x } 10^{-15} \text{ m.}$$

$$Now\ density\ of\ the\ nucleus = \rho_N = \frac{12 \times 1.67 \times 10^{-27}}{\frac{4}{3}\pi(2.7 \times 10^{-15})^3}$$

$$= 2.43 \times 10^{17}\ kgm^{-3}$$

(b) (i) Charge. As the atomic number Z = 82, the charge on the nucleus is

$$+Ze = 82 \times 1.6 \times 10^{-19} \text{ C} = 131 \times 10^{-19} \text{ C.}$$

(ii) Spin. The mass number A = 208. Thus, there are 82 protons 208 - 82 = 126 neutrons. As the nucleus has an even number of particles it has an integral spin.

(iii) Size. The radius of the nuclei is given by

$$r = r_o A^{1/3}$$

where A is the atomic number. In this case A = 208. If r_o, = 1.4 × 10^{-15}m.

Radius of Pb^{208} nucleus r = $r_o A^{1/3}$ = 1.4 × 10^{-15} × $(208)^{1/3}$

= 1.4 × 10^{-15} × 5.926 = 8.296 × 10^{-15} m

= 8.296 fm.

Q. 1.9. The observed spin of $_7N^{14}$ is $\hbar$. Show that it cannot be explained on the electron proton model of the nucleus.

Ans. Spin of $_7N^{14}$. The proton as well as the electron are Fermions having a spin $\frac{1}{2}\hbar$. The resultant spin angular momentum of the nucleus is given by the vector sum of the spin of all the particles in it. If the nucleus has even number of particle it has an integral spin but if it has an odd number of particles then it has a half integral spin. If we accept the electron-proton hypothesis, then there should be 14 protons and 7 electrons in it to give the nucleus a net positive charge of + 7e. As the total number of particles is 14 + 7 = 21 (an odd number) within the nucleus it should have half integral spin. But experimentally the spin is found to be $\hbar$ and not $\frac{1}{2}\hbar$ as required by the theory. Thus electron proton model cannot explain the spin of $_7H^{14}$ to be equal to $\hbar$.

Q. 1.10. (a) A nucleus $_ZX^A$ splits into two fragements

$$_{z_1}Y^{A_1} + \ _{z_2}Y^{A_2}$$

find the separation between the fragments at the moments of their sparation.

Question. Explain the term atomic mass unit. Compute the energy of 1 a.m.u. in MeV.

Q. 1.11. Explain the terms, mass defect, packing fraction, binding energy of a nucleus and binding enemy per nucleon. Sketch and explain the variation of packing fraction with mass number. What is the difference between mass defect and packing fraction?

Ans. (i) Mass defect. The nucleus is formed by bringing protons and neutrons together. The mass of the nucleus so formed is less than the sum of the masses of the constituent protons and neutrons. This mass difference is called mass defect and is denoted by Δm.

If Z is the number of protons in the nucleus (Atomic number Z), then the number of neutrons in the nucleus is (A - Z) (A is atomic mass). If m_p is the mass of the proton and m_n that of the neutron, then

Sum of the masses of the protons and neutrons

$$= Zm_p + (A - Z)m_n$$

If M_N is the actual mass of the nucleus, then

Mass defect $\Delta m = Zm_p + (A - 2)m_n - m_N$

Mass defect is therefore, defined as the difference between the sum of the rest masses of the nucleons forming the nucleus and the actual rest mass of the nucleus.

For example in the case of deutron which contains one proton and one neutron the combined mass is [1.0073 + 1.0087] = 2.1060 a.m.u., whereas the actual mass of deutron nucleus is 2.0136 a.m.u.

∴ Mass defect Δm = 2.1060 - 2.0136 = 0.0024 a.m.u.

(ii) Packing fraction. It has been found that atomic masses, though very close to whole number values invariably differ from the integral value by a small amount. For example the atomic masses of H^1, He^4 and Li^6 are H^1 = 1.007825, He^4 = 4.002603, Li^6 = 6.015126.

The deviation of atomic mass from whole number value is expressed in the form of a quantity known as packing fraction. Packing fraction is defined as the ratio of the difference between the atomic mass of the atom M and its mass number A i.e., (M - A) to its mass number A.

$$\therefore \text{Packing fraction } f = \frac{M-A}{A}$$

Variation of packing fraction with mass number. A graph between packing fraction f (multiplied by a factor 10^4) and mass number A is shown in Fig. 2. It is known as packing fraction curve.

From the graph we find that:

1. For very light nuclei the packing fraction f is maximum indicating thereby that these nuclei are unstable.

2. As the value of A increases packing fraction goes on decreasing (with the exception of ${}_2He^4$ and ${}_6C^{12}$) till it becomes zero for A = 16. Thus, the packing fraction for ${}_8O^{16}$ is zero.

3. As the mass number A increases beyond 16 the value off goes on decreasing further and becomes negative. This continues approximately upto A = 180 (Ta, Tantalum) beyond which f again becomes positive.

4. The value off is minimum for iron (${}_{26}Fe^{56}$) which has a significance as the most stable nucleus.

5. Greater part of the curve is below zero line with negative values of packing fraction indicating that there is loss of mass and hence liberation of energy due to packing.

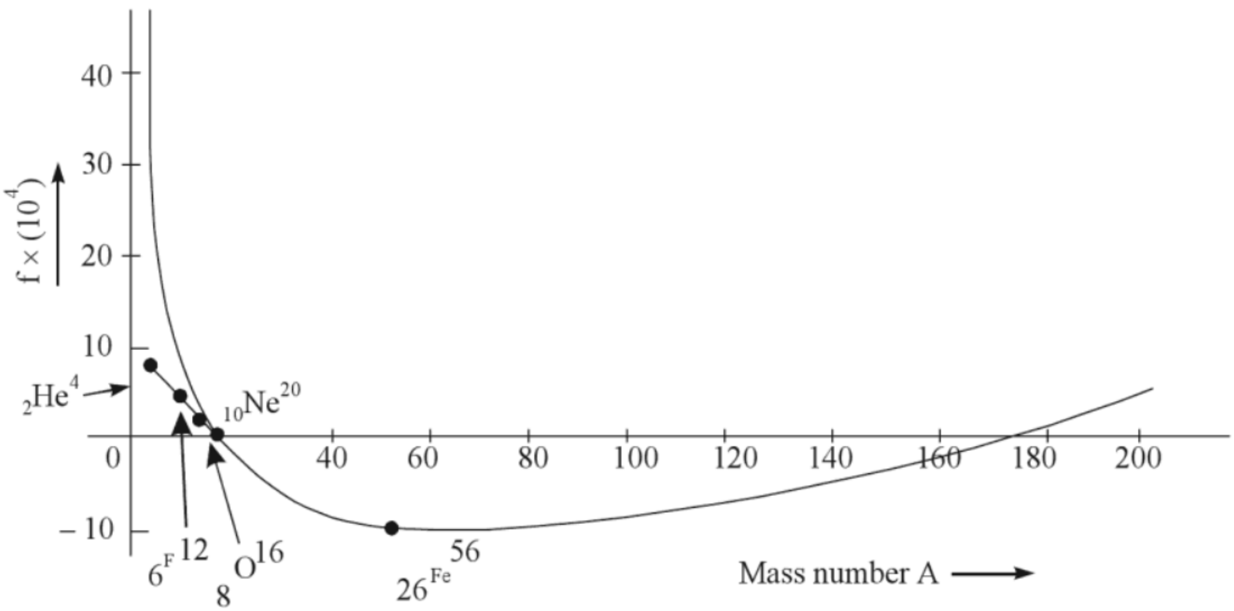


Figure 2

6. For heavy nuclei beyond A = 180 packing fraction is positive. Thus we conclude that nuclei of the middle range are stable while light (below A = 20) and heavy (beyond A = 200) are unstable. All nuclei have a tendency to move from a region of higher packing fraction to lower packing fraction to attain higher stability. This is why fusion of light nuclei and fission of heavy nuclei takes place with a release of energy.

Binding energy. The mass of any permanently stable nucleus is found to be less than the sum of the masses of the neutrons and protons which it contains. The fact is accounted for by the conversion of a part of the mass energy of the particles into energy of binding, the relation between the change in mass and binding energy being given by Einstein's equation

$$\text{Binding energy} = E_B = \Delta mc^2$$

the total binding energy for all atoms except the lighter atoms is given approximately by the empirical relation

$E_B = 15.6\ Z$ MeV

The binding energy of the nucleus with charge number Z and mass number A is given by

$$E_B = C^2\,[Zm_p + (A - Z)m_n - M_N] \ \dots \text{(i)}$$

where M_N is the mass of the nucleus, m_p the mass of the proton and m_n the mass of the neutron. The nuclear mass is, therefore, given by

$$M_N = Zm_p + (A - Z)\,m_n - E_B/c^2$$

and the atomic mass $M = M_N + Zm_e = Zm_e + Zm_p + (A - Z)\,m_n - E_B/c^2$

$$= ZM_H + (A - Z)\,m_n - E_B/c^2$$

where m_e is the mass of the electron and M_H the mass of hydrogen atom. The term E_B/c^2 represents the mass equivalent of the total binding energy i.e., the energy which must be added to the nucleus in order to break it up into Z protons and (A - Z) neutrons.

Binding energy of a nucleus is, therefore, defined as the energy which must be supplied to the nucleus to break it into its constituent nucleons.

Since the mass of a nucleus is less than the sum of the masses of the total number of protons and neutrons in it, the binding energy is a positive quantity. The experimental value of nuclear binding energy varies from 2.23 MeV for deutron, the lightest stable atom containing more than one nucleon to 1640 MeV for the heaviest stable nucleus $^{209}Bi_{83}$.

Difference between mass defect and packing fraction. Packing fraction

$$f = \frac{M - A}{A}$$

where M is the actual mass (or weight) of the nucleus and A the mass number. But M - A = Δm = mass defect

$$\therefore f = \frac{\Delta m}{A} = \frac{mass\ defect}{mass\ number\ (or\ number\ of\ nucleons)}$$

Hence packing fraction is equal to mass defect per nucleon.

Binding energy of the nucleus = Δmc^2 and binding energy per nucleon = $\frac{\Delta m}{A}c^2 = fc^2$

∴ Binding energy per nucleon = c^2 × packing fraction

Binding energy per nucleon is explained in following question

Q. 1.12. (a) Explain the term, binding energy per nucleon and separation energy.

(b) Discuss graphically the variation of average binding energy per nucleon with mass number and stability of the nucleus. Using the binding energy curve, explain the release of energy in fusion of light nuclei and fission of heavy nuclei.

Ans. (a) Binding energy per nucleon or Average binding energy. The average binding energy is defined as binding energy per nucleon. It is the energy required to release a nucleon from the nucleus. From relation Binding energy per nucleon = $\frac{E_B}{A}$

$$\therefore \quad \frac{E_B}{A} = \frac{c^2}{A}\left[Zm_p + (A - Z)m_n - M_N\right] = \frac{\Delta mc^2}{A}$$

$$= c^2\left[m_n - \frac{Z}{A}(m_n - m_p) - M_N/A\right]$$

Separation energy. The separation energy is the minimum energy that must be supplied to remove the least tightly bound nucleon from the nucleus i.e. it is the binding energy of the least tightly bound nucleon.

Graph between binding energy per nucleon and mass number. For a deutron the average binding energy per nucleon = $\frac{2.23}{2}$ – 1.11 MeV. Thus, it is a very loosely bound system. If we plot a graph between E_B/A i.e., binding energy per nucleon and the nucleon number A, the curve Obtained is shown in figure 3. We shall now discuss the variation of average binding energy per nucleon with mass number and stability of the nucleus.

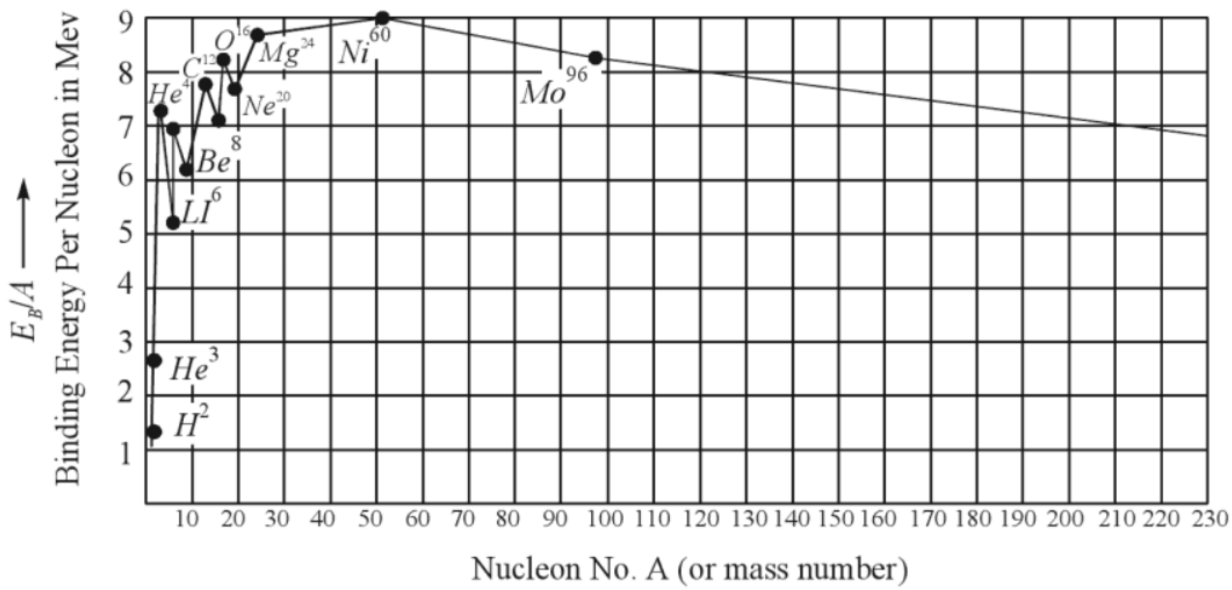


Figure 3

Binding energy and nuclear stability. The binding energy of a nucleus gives a quantitative measure of its stability. Consider the nuclide $_2He^4$ which is made up of two protons and two neutrons for which the binding energy comes out to be 28.299 MeV. In other words, the energy required to break up a $_2He^4$ nucleus (α-particle) in its constituent nucleons we must spend about 28 MeV of energy. Thus the binding energy per nucleon for $_2He^4$ nucleus is about 7 MeV. Greater the binding energy, more stable is the nucleus. We also find that for nuclides like $_2He^4$, $_4Be^8$, $_6C^{12}$, $_8O^{16}$, $_{10}N^{20}$ and $_{12}Mg^{24}$ there is a sharp increase in binding energy. For these nuclides A = 4n = 1, 2, 3, 4, ... etc. All these nuclides can be built up from α-particles, which as proved above, is a very highly bound system.

There is a broad maximum in the mass number range 40 to 120 at an average value of 8.5 MeV. The maximum value of binding energy per nucleon = 8.8 MeV for mass number A very nearly equal to 60. Thereafter, the binding energy per nucleon falls slowly to 7.4 MeV.

It is this diminution in binding energy per nucleon which is mainly responsible for the release of energy in the fission of nuclei. The energy released in fission is several million times that released in chemical changes, since the binding energies of nucleus involved in the process of fission are much higher than the binding energy of electrons involved in the chemical process Similarly, if two very light nuclei are made to fuse into a single nucleus, a large amount of energy will be released. This is known as nuclear fusion.

Nuclides beyond A = 238 will have even smaller binding energy per nucleon and hence are less stable.

The fact that the total binding energy of a nucleus is positive is not a sufficient condition for its stability. For a nucleus to be stable its mass should be less than the sum of the masses of any other combination of its constituent protons and neutrons whether free or bound in small groups. The energy required to remove any given particle from the nucleus is called its separation energy. Thus the separation energy of an α-particle E (α) is given by

$$\frac{E(\alpha)}{c^2} = M({}_2He^4) + M(Z-2, A-4) - M(Z, A)$$

where M (${}_2He^4$) is the mass of the α-particle, $M(Z-2, A-4)$ the mass of the nucleus having (Z -2) protons and mass number (A - 4) and M (Z, A) the mass of the nucleus having Z protons and mass number A. If E(α) is positive the nucleus is stable to α-emission, e.g., ${}_{10}Ne^{20}$ for which E(α) = +4.730 MeV. On the other hand, if E (α) is negative, the nucleus is unstable to α-emission e.g., ${}_{84}Po^{210}$ for which E (α) = -5.4 MeV is α-radioactive.

Sometimes it is possible that a nucleus may have insufficient energy to emit a nuclear particle but a system of lower energy may be formed by changing a neutron into a proton and vice-versa, with the emission of a β^- or β^+ particle. A nuclide is stable to negative β decay if

$$M(Z, A) \leq M(Z+1, A) + m(\beta^-)$$

and for stability to positive β decay, the condition is

$$M(Z, A) \leq M(Z-1, A) + m(\beta^+)$$

A third type of β process is also possible and is known as decay by electron capture. In this process an orbital electron, usually from the K-shell, is absorbed into the nucleus and a neutron is emitted which may have zero energy. The corresponding condition for stability is

$$M(Z, A) \leq M(Z-1, A)$$

Release of nuclear energy in fission and fusion. From the graph between binding energy per nucleon and mass number we find that the value of binding energy per nucleon is small both for very light and very heavy nuclei so that these substances are unstable. The nuclei of intermediate masses (mass number 40 to 120) are the most stable and very high amount of energy has to be supplied to liberate each of their nucleons.

It, therefore, follows that when a heavy nucleus breaks up into lighter ones (nuclear fission) the end products formed have higher value of binding energy thereby resulting in the liberation of energy. Similarly, when two very light nuclei combine to form a heavy nucleus (nuclear fusion) the higher binding energy per nucleon of the latter again results in the liberation of energy.

Question. Which is more, atomic binding energy or nuclear binding energy?

Ans. Nuclear binding energy is much more than atomic binding energy. For example, the binding energy per nucleon of α-particle is of the order of 7 MeV whereas the binding energy of the orbital electron in hydrogen atom is only 13.6 eV.

$$\therefore \frac{Nuclear\ binding\ energy}{Atomic\ binging\ energy} = \frac{7 \times 10^6}{13.6}$$

$$= 5.1 \times 10^5$$

Q. 1.13. Prove from wave mechanical, angular momentum, statistical and other considerations that electrons cannot exist in the nucleus.

Q. 1.14. (a) Calculate the binding energy per nucleon for the deutron. Given $m_n = 1.675 \times 10^{-27}$ kg, $m_p = 1.672 \times 10^{-27}$ kg; $M_D = 3.343 \times 10^{-27}$ kg; $c = 3 \times 10^8$ ms^{-1}.

(b) Calculate the binding energy of an α-particle from the following data

Mass of helium nucleus = 4.002870 a.m.u.

Mass of proton = 1.007825 a.m.u.

Mass of neutron = 1.008665 a.m.u.

Explain why the sum of the masses of 2 neutrons and 2 protons is not equal to the light helium isotope.

(c) Compare the specific binding energy of tritium and of the light helium isotope. Given Mass of proton = 1.0073 a.m.u.; Mass of neutron = 1.0087 a.m.u.; Mass of tritium = 3.016 a.m.u. and mass of $_2He^3$ = 3.016a.m.u.

Q. 1.15. Find the energy released if two $_1H^2$ nuclei can fuse together to form the $_2He^4$ nucleus. The binding energy per nucleon of H^2 and He^4 is 1.1 MeV and 7.0 MeV. m_n = 1.008665 a.m.u. and m_p = 1.007825 a.m.u.; $_2He^4$ = 4.002603.

Q. 1.16. (a) What is the energy of an electron at rest?

(b) Calculate the ratio of $\frac{m}{m_o}$ for electron having kinetic energy 1MeV. Letters have their usual meaning. $m_o = 9.1 \times 10^{-31}$ kg.

Q. 1.17. (a) The mass of Lithium atom is 7.01822 a.m.u. Calculate the binding energy (in electron volts) of $_3Li^4$ nucleus. Given mass of proton = 1.00814 a.m.u. mass of neutron = 1.00893 a.m.u. mass of electron = 0.00055 a.m.u.

(b) Calculate the average bonding energy per nucleon for $_{28}Ni^{64}$ having 63.9280 a.m.u. Given that Z = 28, A =64, m_p = 1.007825 a.m.u. and m_n = 1.008665 a.m.u.

Q. 1.18. Calculate binding energy per nucleon for $_{17}Cl^{35}$

m_p = 1.007825 a.m.u.

m_n = 1.008665 a.m.u.

M_N = 34.9800 a.m.u.

Q. 1.19. The binding energy of $_{10}Ne^{20}$ is 160.64 MeV. Find the atomic mass. Given mass of proton = 1.007825 a.m.u. and of neutron = 1.008665 a.m.u.

Q. 1.20. Given $_{10}Ne^{20}$ is 19.9924 a.m.u. Using conversion factor obtain its binding energy in MeV.

Q. 1.21. (a) Explain the nuclear forces responsible for holding the nuclear together. Discuss the nature of these forces giving a qualitative account.
(b) what do you mean by charge independence of nuclear forces?
Ans. (a) Nuclear forces. The radius r_n of the nucleus of mass number A is given by

$$r_n = r_o A^{1/3}$$

where r_o is a constant for all nuclei and has a value 1.4×10^{-15} m. Hence the nuclear mass and nuclear volume are both proportional to A. Therefore, the nuclear density is nearly constant for all nuclides.
The nucleus consists of protons and neutrons. As the protons carry a positive charge, the electrostatic force of repulsion between them should cause a disruption of the nucleus. The gravitational force of attraction between the neutrons is too weak to account for the observed binding energy of the nuclei. Thus, there must be some other forces which bind the nucleons. As there are two types of nucleons, the proton and the neutron. There are three types of attractive forces acting in the nucleus. These are:

(i) Force between two protons ($p - p$ force)
(ii) Force between a proton and a neutron ($p - n$ force)
(iii) Force between two neutrons ($n - n$ force)

***p – p* force.** The ($p - p$) force has been examined by proton-proton scattering experiment t. These experiments show that the force can be represented by the potential curve shown in Fig.
At large distance of separation the protons repel one another by the Coulomb electrostatic force. At a distance of approximately 3×10^{-15} m a fairly sharp break in the potential curve occurs. At smaller distances the protons strongly attract each other and this strong attractive force is the nuclear forces between pair of protons.
***p – n* force.** The ($p - n$) force has been examined by neutron-proton scattering experiment. These experiments show that the neutron-proton force can be represented by potential curve shown in

Fig. At large distance of separation there is no force between the two particles but at distances of about 2×10^{-15} m the neutron and proton attract one another by a strong nuclear force.

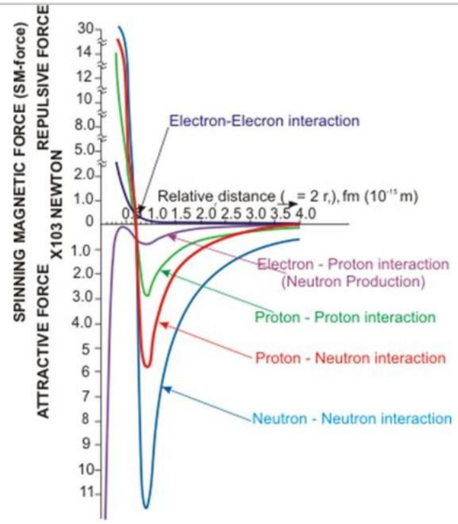


Figure 4: Forces between nucleons.

The existence of ($p - n$) force of attraction is proved by the stability of deutron nucleus consisting of 1 proton and 1 neutron which has binding energy of 2.2 MeV.

We know that the atomic mass number A is approximately equal to twice the atomic number Z for the light and intermediate nuclei. It shows that nuclei prefer to add nucleons in pairs. i.e., There is a strong interaction between the neutrons and protons.

***n* -*n* force.** The neutron excess in heavy nuclei confirms that (n -n) forces are attractive but not sufficiently large to lead to a stable dineutron just as a stable diproton does not exist. The scattering length of (n -n) interaction appears to have the same (negative) sign as the ($p - p$) interaction.

Properties of nuclear forces. (*i*) Nuclear forces are short range forces so that each nucleon comes under the influence of only those nucleons which are in its immediate close vicinity. Nuclear forces are applicable only when the distance between the nucleons is of the order of 2.2×10^{-15}m = 2.2 fm. These forces vanish for all practical purposes at distances greater than about 4.2×10^{-15}m. If these are long range forces, then each nucleon will come under the influence of all other nucleons in the nucleus. In such a case, the binding energy will be proportional to (*i*) the number of nucleons exerting the force and (*ii*) the number of nucleons on which the force is being exerted. Hence the binding energy will be proportional to the square of the number of nucleons in the nucleus i.e., $\propto A^2$. But the binding energy per nucleon is almost a constant and, therefore, the total binding energy of the nucleus is proportional to the number of nucleons i.e., $\propto$ A. On the other hand, if these are short range forces, then each nucleon will come under the influence of only those nucleons which are in its immediate vicinity in the nucleus and these alone will contribute to the binding energy of the nucleons making it almost proportional to A.

(*ii*) Nuclear forces are charge independent. From a study of mirror nuclides, we find that the three types of forces i.e., ($p - p$), ($p - n$) and ($n - p$) are almost of equal magnitude. Mirror nuclides are those nuclides which contain the same number of nucleons but in which proton and the neutron number are interchanged. For example, the nuclide ${}_1H^3$ which contains 1 proton and 2 neutrons and the nuclide ${}_2H^3$ which contains 2 protons and 1 neutron are mirror nuclides. The binding energy of 8.5 MeV of ${}_1H^3$ nucleus is due to 2(*n* - *p*) forces 1(*n* - *n*) force and that of ${}_2H^3$ nucleus is equal to 7.7 MeV is due to 2(*n* - *p*) forces and 1(*p* - *p*) force. The three types of forces being equal in magnitude, the difference of 8.5 – 7.7 = 0.8 MeV in the binding energies of ${}_1H^3$ and ${}_2He^3$ nuclides is due to the fact that the (*n* – *n*) force in ${}_1H^3$ is greater than (*p* – *p*) force in ${}_2He^3$ due to the electrostatic force of repulsion between two protons. If allowance is made for this force, then (*n* – *n*) and (*p* – *p*) forces are equal.

The equality of (*n* – *n*), (*p* – *p*) attractive forces is also established from a study of other mirror nuclei after making allowance for the electrostatic force of repulsion between 2 proton and it is found that total binding energy depends only on the total number of nucleons and not on their nature. In other words, nuclear forces are charge independent. This conclusion is also confirmed by other experiments like scattering of proton by hydrogen nuclei and neutrons by neutrons.

It, therefore, follows that nuclear forces are of non-electric nature.

(iii) Nuclear forces are the strongest known forces is nature. The magnitude of nuclear forces is many times the electrostatic repulsive force between the proton and about 10^{38} times the gravitational force between the neutrons.

(iv) Nuclear forces are spin dependent. It has been observed that the force of attraction between two nucleons having parallel (↑↑) spin is stronger than the force between two nucleons having anti-parallel (↑↓) spin

(v) Nuclear forces have property saturation. Nuclear forces are limited in range. As a result each nucleon interacts only with a limited number of nucleons nearest to it. This effect is known as saturation of nuclear forces.

The saturation of nuclear forces arises from the dependence of total binding energy on the mass number A. if there were no saturation and each nucleons could interact with the remaining (A-1) nucleons, then the total binding energy would be proportional to the number of nucleon pairs in the nucleus i.e., $\frac{A(A-1)}{2}$, in other words binding energy will be proportional to A^2 , but the binding energy is proportional to A. Hence nuclear force are saturated forces.

(vi) Nuclear forces are non-central. The force existing between the two nucleons has a non-central component that does not point along the line joining the two nucleons. This non-central component depends upon how the nuclear spins are oriented relative to the line joining the nucleons.

(vii) Nuclear forces are exchange forces. Nuclear forces have an exchange character as these forces are brought into the existence due to the change of π^+,π^- and π^0 mesons (pions) between the nucleons.

(viii) At the close distance of approach between the nucleons, nuclear forces become force of repulsion.

(b) Charge independence of nuclear forces is described in part (a)

INTRODUCTION TO NUCLEAR REACTIONS

Continuous research on artificial transmutation and especially the study of induced radioactivity, culminated in the discovery of nuclear fission which is accompanied by the release of enormous amounts of energy. In ordinary nuclear disintegrations, both neutral and artificial, the nucleus is only chipped off rather than broken and accordingly, the amount of energy released is comparatively less i.e. from about 10 to 23 MeV.

It was discovered in 193 that the heavy unstable Uranium nucleus when bombarded by neutrons splits into two almost equal fragments which fly apart with great speed and the amount of energy released per fission is about 200 MeV.

This division of a nucleus into two approximately equal parts is called nuclear fission.

DISCOVERY OF FISSION

The starting point in the discovery of nuclear fission can be traced to the attempts of fermi, in 1934, to produce transuranic elements by bombarding uranium with neutrons. However, the fission process itself was discovered in 1939 by German radio-chemists Otto Hahn and his two associates Meitner and Strassmann. After bombarding uranium with neutrons, they performed a series of chemical separations to identify the products. To their great surprise, they found that the atoms produced by the bombardment of uranium belonged to elements which lie in near the centre of the periodic table. Obviously, a uranium nucleus after capturing neutron had become so unstable that instead of disintegrating by ejecting one or two particles, it had split up into two parts.

The actual fission process can be understood with the help of figure. 11.1 which shows a uranium nucleus capturing a neutron.

The newly-formed nucleus of fig 11.1(b) is unstable and starts breaking up into two parts. In breaking up, the uranium nucleus, behaving like a liquid drop, splashes out small droplets i.e. neutrons and - rays. So, great is the release of energy that the two fission fragments fly apart in opposite directions with tremendous speeds. It may, however, be noted that not a uranium nuclei break into Sb and Nb as shown in figure 11.1. There are at least 30 different ways in which a fissile nuclide can divide itself. The experimental (asymmetric fission) accompanied by one to five or some more neutrons. In general, fission fragments are unstable nuclei containing an access number of neutrons. After a series of -emissions in which neutrons are converted into proton in the nucleus, a stable nuclide results (fig. 11.1).

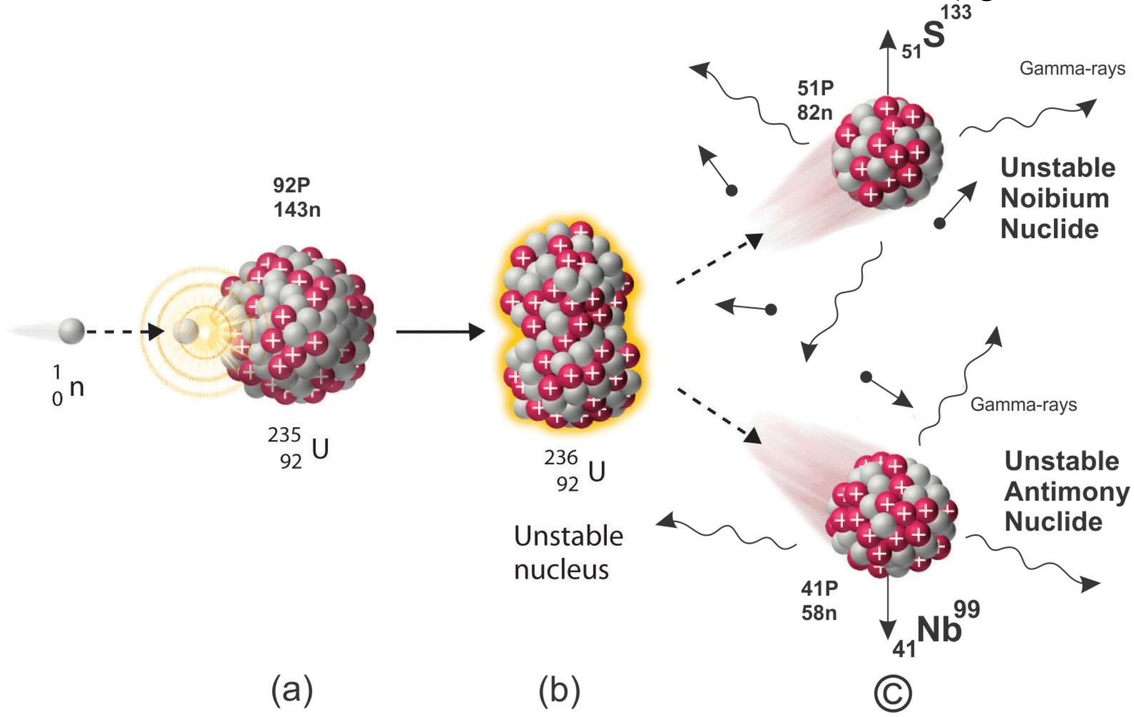


Fig. 11.1

Out of all, the neutrons ejected during the fission of Uranium, about 99 percent are ejected in an extremely short interval of time and are called prompt neutrons. The remaining one percent of neutrons are emitted a little later and called delayed neutrons. The delayed neutrons originate from unstable fragments that decay by neutron emission on their way to becoming stable nuclei.

It may be noted that division of fissile nucleus into three fragments of comparable sizes (ternary fission) has been observed although it is a rare event, occurring about 5 times per million binary fissions.

Types of fission reactions

Historically speaking, uranium was the first element to undergo fission. However, soon after it was found that other elements of high atomic weight could also be made to undergo fission and that particles other than neutron could be equally effective in this respect.

Natural uranium contains three principal isotopes with the following relative abundance:

U^{238}	99.388	4.51×10^9 Y	99.2739%
U^{235}	0.7148	7.1×10^8 Y	0.7205%
U^{234}	0.0068	2.48×10^5 Y	0.0056%

It is found that slow neutrons cause fission of U^{235} but not of more abundant isotope U^{238} which requires fast neutrons with energies exceeding 1 MeV. Similarly, Th^{232} and Pa^{231} undergo fission when bombarded with fast neutrons. Fission can also be produced in uranium and thorium by high-energy -particles, protons and deuterons and -ray etc. Two other nuclides which do not occur in nature but have proved to be fissionable by neutrons of all energies are $_{92}U^{233}$ and $_{94}Pu^{239}$. In 1947, successful fission of Bismuth, Lead, thallium, mercury, gold, platinum, tantalum was achieved in USA by means of -particles, deuterons and neutrons of 100 MeV and more. With Bismuth (Z = 83) fission was detected with 50 MeV deuterons whereas tantalum (Z = 73) required -particles of 400 MeV energy.

It is worth nothing that only three fissile materials U^{233}, U^{235} and Pu^{239} are important in the large-scale application of nuclear energy.

Finally, some heavy nuclei have been found to undergo spontaneous fission. In this process, nucleus divides in the ground state without bombardment by particles from outside.

Mass distribution of fission products

During uranium fission, a large number of nuclides of intermediate charge and mass are found. Their study is a promising source of information about the mechanism of the fission process itself and also offers the possibility of discovering hithero unknown nuclides. Investigations of the fission products of U^{235} have shown that the range of their mass numbers is from 72 to 158. About 97% of U^{235} nuclei undergoing fission give fragments which fall into two groups as shown in the fission yield curve of Fig. 11.2.

a) Light group with mass numbers from 85 to 104 and

b) Heavy group with mass numbers from 130 to 149.

The most probable type of fission which occurs in about 7% of the total cases, gives fission products with mass numbers 95 and 139.

As mentioned earlier, fission fragments have too many neutrons in their nuclei for stability. Consequently, most of them decay by electron emission. Each fragment starts a short radio-active series involving many emissions of β - particles. These series are called fission decay series

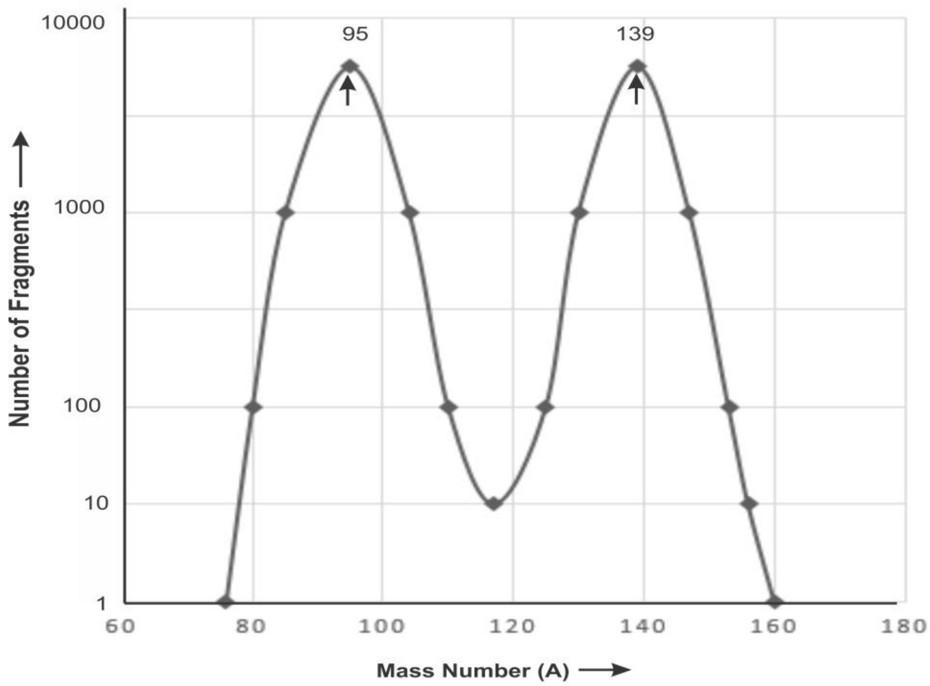


Fig. 11.2

and each chain has three members on the average although a longer and shorter chains occur frequently. One

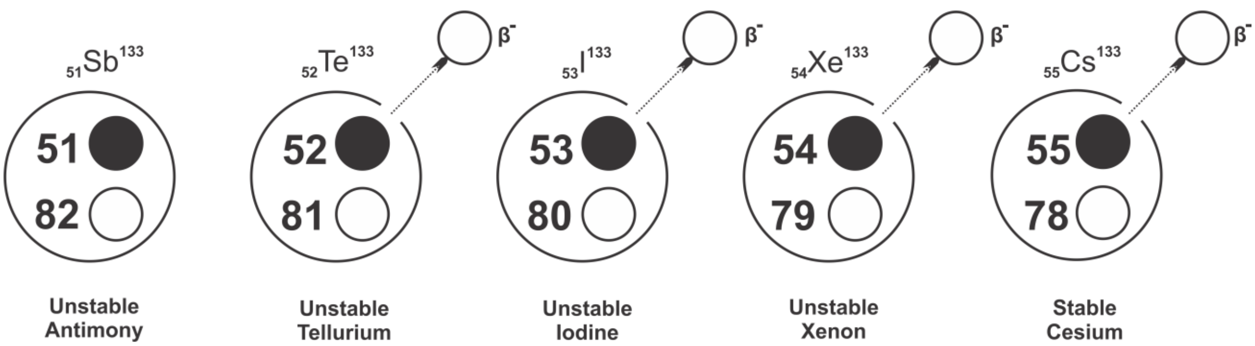


Fig. 11.3

such fission decay chain is shown in Fig. 11.3 which starts with one of the unstable fragments of the fission of U^{235} nucleus.

Energy distribution of fission products

Energy distribution among the ioucts can be found by measuring their kinetic energy with the help of suitable ionization chamber. The results of such study on U^{235} fission have shown that the energy distribution curve is not uniform; rather it is a double-peaked curve with maximum at 67 MeV and 100 MeV. It is seen that while the greatest probability is for a fragment of 100 MeV, the areas under the two peaks which represent the total number of particles in the two groups are approximately equal.

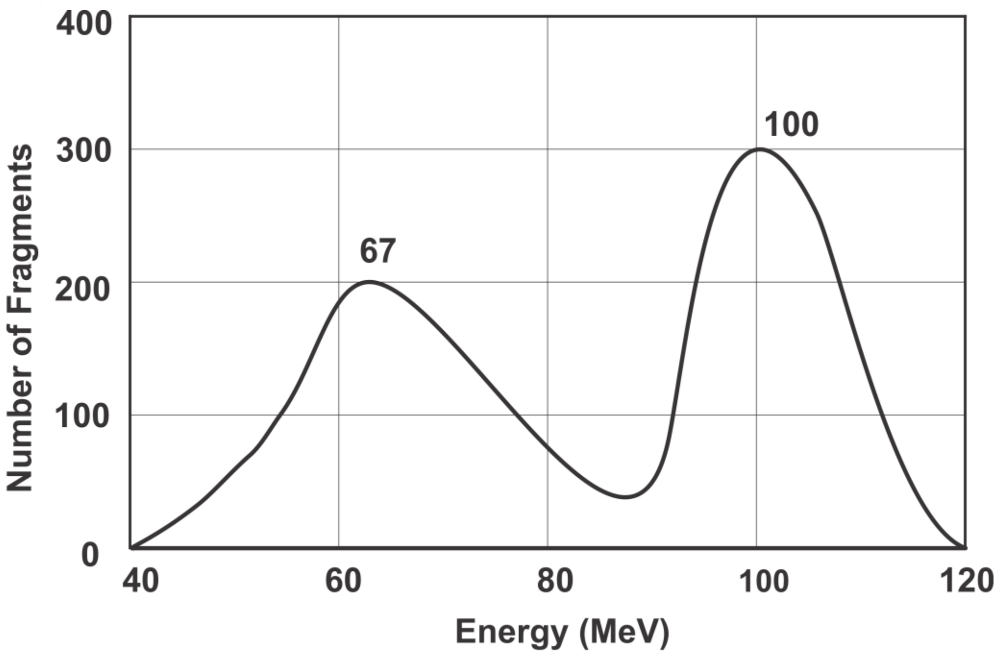


Fig. 11.4

Neutron emission in nuclear fission

One of the notable features of nuclear fission is that while it is initiated by neutrons, it is also accompanied by the emission of fast-moving neutrons. The number of neutrons released depends on the mode of fission and on the energy of the neutrons which induce fission. The average values for the number of neutrons emitted per thermal neutron absorbed by the three important fissile materials are given below:

U^{235}	2.43
U^{233}	2.50
Pu^{239}	2.89

These neutrons are emitted by the fission fragments and not by the compound nucleus.

The neutrons emitted as a result of fission process (i.e. Fission neutrons) can be divided into two groups.

i. **Prompt Neutrons.** These make up about 99.36% of the total fission neutrons and are ejected by the product nuclei within 10^{-10} second of the fission process. Prompt γ-rays are also emitted at the same time.

ii. **Delayed Neutrons.** These constitute about 0.64% of the total neutrons from the fission of U^{235}. These are emitted with gradually decreasing intensity for several minutes after actual fission process. Although the number of delayed neutrons is small, they have a strong influence on the time dependent behaviour of chain-reacting systems based on fission and play an important role in the control of nuclear fission reactors.

Sources of neutrons

Neutrons are the most important of the fundamental particles as they readily enter nuclei and induce transformations. They are much valuable for experimental purposes. For use in experiments there are four chief methods for their production.

a) A weak source of neutrons is obtained from a Beryllium target bombarded by α-particles in which neutrons are produced by the reaction.

$$_4Be^9 + {}_2He^4 \longrightarrow {}_6C^{12} + {}_0n^1$$

The neutrons produced have a wide range of energies. A radium-beryllium source using one mg of radium emits about 10^4 neutrons per second with the energies in the range of 5 MeV to 13 MeV.

b) Disintegration of deuterium and beryllium by γ-rays produce a practically monoenergetic sources of neutrons.

c) When accelerated deuteron from a cyclotron or other accelerators impinge a beryllium or deuterium target, neutrons are emitted.

d) The most powerful source of neutron is the nuclear reactor.

Fissile and fissionable nuclides

Elements like U^{235}, U^{233} and Pu^{239} undergo fission by neutrons of energy from almost zero upwards. Such nuclei are referred to as fissile nuclides. On the other hand, U^{238} and Th^{232} nuclei which have a fission threshold at 1 MeV are said to be fissionable nuclides.

In general, fissile nuclides have either an even number of protons and an odd number of neutrons or odd number of both. Fissionable nuclides on the other hand, have either even number of protons or neutrons or an odd number of protons and an even number of neutrons.

Characteristic features of nuclear fission

Detection of fission

The fission process may be detected by the following means.

i. Chemical identification of the products as in the original observation of Hahn and Strassmann.

ii. The fission fragments may be caught upon a receiving surface placed close to the irradiated material and detected sub-sequently by radioactivity (Joliots)

iii. **<u>Ionization chamber method.</u>** coating the walls of the ionization chamber with the fissionable material and allowing the projectile to bombard the walls, huge pulses of ionization produced by the fission fragments can be observed.

iv. **Cloud Chamber Method.** Heavy tracks made by fission fragments in a cloud chamber may be photographed.

v. **Photographic Plate Method.** The fissionable material is introduced into the plate by a bathing process using a solution of U-citrate. When the plate is exposed to slow neutron bombardment, heavy tracks of fission fragments are recorded.

Other particles inducing fission

Fission has been effected in both U and Th by 6.9 MeV protons and by deuterons of energy greater than 8 MeV and by α-particles of energy 32 MeV. High energy γ-rays obtained from nuclear reactor or X-ray from a Betatron have also been effective in producing fission.

Fission energy

One of the striking features of the fission process is the magnitude of energy released which is about 200 MeV per fission of U^{235} nuclide. Before 1939, the largest known nuclear reaction energy was 22.2 MeV associated with Li^6 (d, α) He^4 reaction.

The amount of energy released per fission of U^{235} nuclide may be calculated by the following three methods.

Binding-energy method

All stable fission products have mass numbers in the range 72 to 158 where the average binding energy per nucleon is about 8.5 MeV. However, in the neighborhood of uranium, its value is 7.6 MeV. Hence, average binding energy per nucleon is (8.5 - 7.6) = 0.9 MeV greater in the fission products than in the compound nucleus of U^{235}. The excess energy is released as fission energy. Its value is = 235 × 0.9 = 200 MeV per fission of U^{235} nuclide (when has 235 nucleons).

Mass defect method

The energy released per fission can also be estimated by comparing the mass of interacting particles and the final fission products.

U^{235} splits in many ways and the nuclei obtained in the greatest yield in fission by slow neutrons have mass numbers of 95 and 139. The fission products being initially radioactive, undergo many emissions to form ultimately stable nuclides. If molybdenum-95 and lanthanum-139 are taken as pair of stable products from fission of U^{235}, the fission reaction can be written as

$$_{92}U^{235} + {}_0n^1 \rightarrow [_{92}U^{236}] \rightarrow {}_{42}Mo^{95} + {}_{57}La^{139} + 2\,{}_0n^1$$

Comparing masses of both sides of the above equation, we get

Mass of U^{235} nuclide = 235.124 amu

Mass of one neutron = 1.009 amu

Total	=	236.133 amu
Mass of Mo^{95} nuclide	=	94.946 amu
Mass of La^{139} nuclide	=	138.955 amu
Mass of two neutrons	=	2.018 amu
Total	=	235.919 amu
Mass defect	=	236.133 - 235.919
	=	0.214 amu

Therefore, energy released per fission of U^{235} nucleus = 0.214 × 931 = 200 MeV

Kinetic energy measurement method

The total amount of energy released per fission is equal to the sum of the following kinetic energies:

a) The kinetic energy of the fission fragments. As seen from Fig. 11.4, the average value of this energy for U^{235} is 167 MeV.

b) The kinetic energy of fission neutrons. Since the average number of neutrons emitted per fission of U^{235} is 2.43 or say 2.5 and the average kinetic energy of these neutrons is 2 MeV, total kinetic energy of fission neutrons is = 2.5 × 2 = 5 MeV.

c) The kinetic energy of prompt γ-rays. Its value is about 7 MeV.

d) Total energy of the decay processes in the fission decay chains. This include the energy carried away by radiations like β - rays, γ-rays and neutrons. Its value is nearly 21 MeV.

The total of all the above energies is = 167 + 5 + 7 + 21 = 200 MeV.

Chain reaction

A chain reaction is a self-propagating process in which number of neutrons goes on multiplying rapidly almost in geometrical progression during fission till whole of fissile material is disintegrated.

Example: suppose a simple neutron causing fission in a uranium nucleus produces 3 prompt neutrons. The three neutrons in turn may cause fission in three uranium nuclei producing 9 neutrons. These 9 neutrons in turn may cause fission in nine uranium nuclei producing 27 neutrons and so on. The number of neutrons produced in such generations is 2n. The ration of secondary neutrons produced to the original neutrons is called the multiplication factor (k).

Consider 1 Kg of $_{92}U^{235}$ which contains 6.02 × 10^{26} / 235 or about 25 × 10^{23} atoms. Suppose a stray neuron causes fission in a uranium nucleus. Each fission will release on the average 2.5 neutrons. The velocity of a neutron among the uranium atoms is such that a fission capture of a thermal neutron by the $_{92}U^{233}$ nuclei takes place in about 10^{-8} s. Each of these fissions, in turn, will release 2.5 neutrons. Let us assume that all these neutrons are available for inducing further fission reactions. Let n be the number of stages of fission disrupt required to disrupt the entire mass of 1 kg of ${}_{92}U^{235}$.

Then

$$(2.5)^n = 25 \times 10^{23} \quad \text{or} \quad n \approx 60$$

The time required for 60 fissions to take place = 60×10^{-8} s = 0.6 μs.

Since each fission releases about 200 MeV of energy, this means that a total of $200 \times 25 \times 10^{23} = 5 \times 10^{26}$ MeV of energy is released in 0.6 μs. The release of this tremendous amount of energy in such a short time interval leads to a violent explosion. This results in powerful air blasts and high temperature of the order of 10^7 K or more, besides intense radioactivity. The self-propagating process described here is called a chain reaction. Two types of chain reaction are possible. In one, the chain reaction is first accelerated so that the neutrons are built up to a certain level and thereafter the number of fission producing neutrons is kept constant. This is controlled chain reaction.

Such a controlled chain reaction is used in nuclear reactors. In the other type of chain reaction, the number of neutrons is allowed to multiply indefinitely and the entire energy is released all at once. This type of reaction takes place in atom bombs.

Multiplication factor (K). The ratio of secondary neutrons produced to the original neutrons in called the multiplication factor. It is defined as

$$k = \frac{Number\ of\ neutrons\ in\ any\ one\ generation}{Number\ of\ neutrons\ in\ the\ preceding\ generation}$$

The fission chain reaction will be "Critical" or steady when k = 1, it will be building up or "supercritical" when k > 1 and it will be dying down "subcritical" when k < 1.

Critical size for maintenance of chain reaction. Consider a system consisting of uranium (as a fissile material) and a moderator. Even though each neutron that produces fission ejects 2.5 neutrons on an average, all of them are not available for further fission. The maintenance of the chain reaction depends upon a favorable balance of neutrons among the three processes given below:

1) The fission of uranium nuclei when produces more neutrons than the number of neutrons used for inducing fission.

2) Non-fission processes, including the radiative capture of neutrons by the uranium and the parasitic capture by the different substances in the system and by impurities.

3) Escape or leaking of neutrons through the surface of the system.

If the loss of neutrons due to the last two cases is less than the surplus of neutrons produced in the first, a chain reaction takes place. Otherwise it cannot take place.

The escape of neutrons takes place from the surface of the reacting body and fission occurs throughout its volume.

$$\therefore \; Escape\ rate\ varies\ as\ r^2$$

$$(\therefore \; area\ of\ spherical\ mass = 4\pi r^2)$$

and production rate varies as r^3 ($\because \; Volume = \frac{4}{3}\pi r^3$)

$$\therefore \; \frac{Escape\ rate}{Production\ rate} \propto \frac{1}{r}$$

The larger the size of the body, the smaller is the escape rate. Thus it is clear that by increasing the volume of the system, the loss of neutrons by escape from the system is reduced. The greater the size of the system, the lesser is the probability of escape neutrons. In this case, the production of neutrons will be more than the loss due to other causes and a chain reaction can be maintained. Thus, there is a critical size for the system. Critical size of a system containing fissile material is defined as the minimum size for which the number of neutrons produced in the fission process just balance those lost by leakage and non-fission capture. The mass of the fissionable material at this size is called the critical mass. If the size is less than the critical size, a chain reaction is not possible.

Natural uranium and chain reaction. Natural uranium consists of 99.28% of U^{238} and 0.72% of U^{235}. As most of the mass of natural uranium consists of U^{238}, the neutrons released during nuclear fission will try to bombard the nuclei of U^{238} mostly and very few will bombard U^{235}. U^{235} undergoes fission even by neutrons of small energy like thermal neutrons. U^{238} is fissionable only with fast neutrons of energy 1 MeV or more. It has been found that very few neutrons can cause fission of U^{238} but neutrons of all possible energies can cause fission of U^{235}. Thus, chain reaction is not possible in natural uranium.

A chain reaction can, however, be made to develop in natural uranium, if the fast neutrons from it are quickly reduced to thermal ones before they are lost through non-fission capture in the uranium, so that the chances of the thermal neutron fission of U^{235} go up. The neutrons can be slowed down by distributing among lumps or rods of uranium a material called moderator. The moderators must not absorb the neutrons. The function of the moderator is to slow down the neutrons produced by fission by elastic collision. Materials used as neutron moderators have a large inelastic scattering cross-section and, at the same time, a small neutron-capture cross section. Commonly used moderators are graphite, heavy water (D_2O), beryllium, beryllium oxide, hydrides of metals and organic liquids. The nuclei of these substances absorb neutrons only to a slight extent.

Fig. 11.6 shows a self-sustaining chain reaction. A slow neutron bombards a U^{235} nucleus. The nucleus breaks into two fragments and in the fission process three fast neutrons are emitted. The neutrons are slowed down by the moderator. One neutron may be captured by U^{238} to form

U^{239}, which decays to Np^{239} and then to Pu^{239}. One neutron is still available for carrying on the chain reaction. It bombards U^{235} and the process is repeated.

Impacted Nucleus (X)	**Compound Nucleus (c)**	**Conclusion**
Even-even	Even-odd	No fission by thermal neutrons
Even-odd	Even-even	Fission possible by thermal neutrons
Odd-even	Odd-odd	No fission by thermal neutrons
Odd-odd	Odd-even	Fission possible by thermal neutrons

Atom bomb

The principle of fission is made use of in the construction of the atom bomb. An atom bomb consists essentially of two pieces of $_{92}U^{235}$ (or $_{94}Pu^{239}$) each smaller than the critical size and a source of neutrons. The two subcritical masses of U^{235} in the form of hemi spheres are kept apart by using a separator aperture (Fig. 11.7). When the bomb has to be exploded, a third

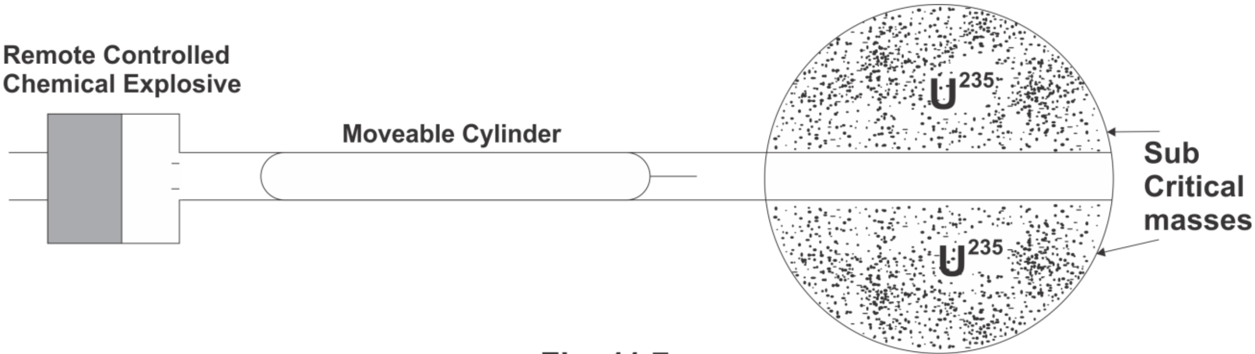


Fig. 11.7

well-fitting cylinder of U^{235} (whose mass is also less than the critical mass) is propelled so that it will fit in or fuse together with the other two pieces. Now the total quantity of U^{235} is greater than the critical mass. Hence an uncontrolled chain reaction takes place resulting in a terrific explosion.

The explosion of an atom bomb released tremendously large quantity of energy in the form of heat, light and radiation. A temperature of millions of degrees and a pressure of millions of atmospheres are produced. Such explosions produce shock waves. They are very dangerous because the waves spread radioactivity in air and cause loss of life. The release of dangerously radioactive γ-rays, neutrons and radioactive materials presents a health hazard over the surroundings for a long time. The radioactive fragments and isotopes formed out of explosion adhere to dust particles thrown into space and fall back to earth causing a radiation "fall-out", even at very distant places.

Nuclear reactors

We know that during the fission of U^{235} a large amount of energy is released. The atom bomb is due to an uncontrolled chain reaction. A very large amount of energy is liberated within an extremely small interval of time. Hence it is not possible to direct this energy for any useful purpose. But in a nuclear reactor, the chain reaction is brought about under controlled conditions. If the chain reaction is put under control, after some time a steady state is established. Under a steady state, the rate of energy production also attains constant level. Such a device in which energy is released at a given rate is known as a nuclear reactor. Nuclear reactors consist of five main elements.

1) The fissionable material called fuel,
2) Moderator,
3) Neutron reflector,
4) Cooling system and
5) The safety and control systems.

1) **The fissionable substance**. The commonly used fissionable materials are the uranium isotopes U^{233}, U^{235}, the thorium isotope Th^{232} and the plutonium isotopes Pu^{239}, Pu^{240} and Pu^{241}.

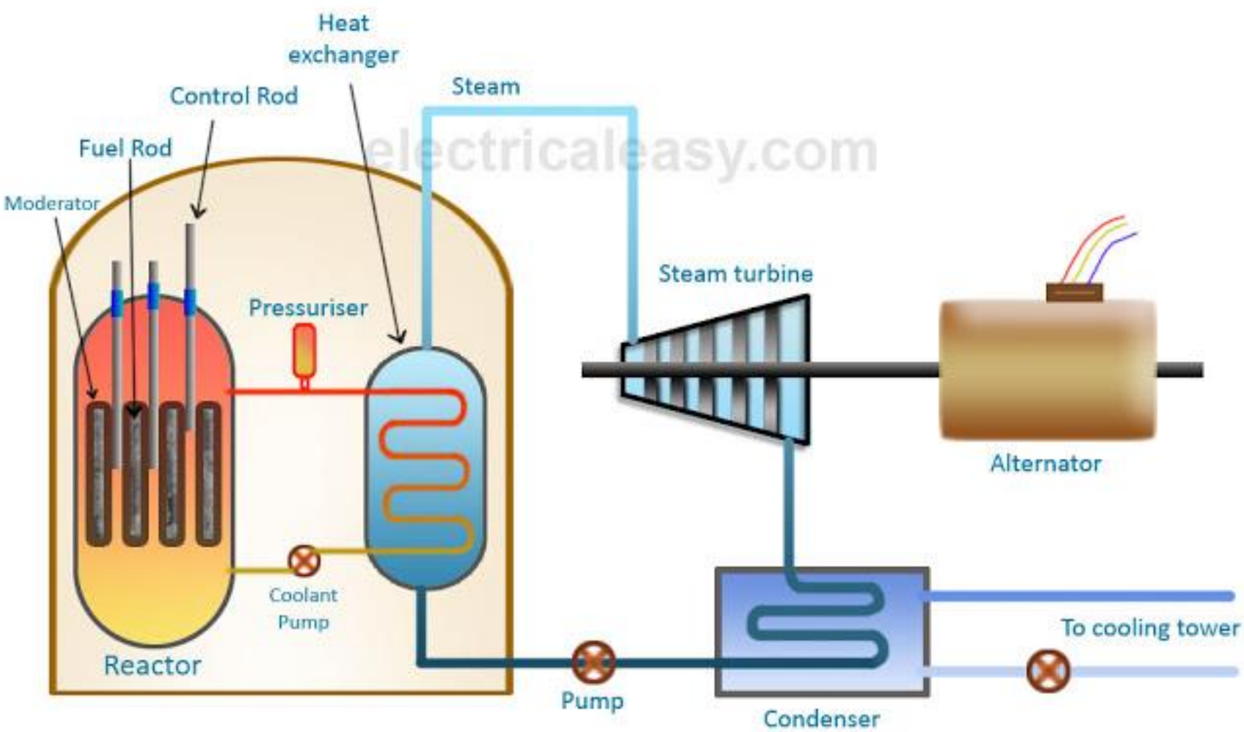
2) **Moderator.** The function of the moderator is to slow down the highly energetic neutrons produced in the process of fission of U^{235} to thermal energies. Heavy water (D_2O), graphite, beryllium, etc. are used as moderators. Ideally, moderators have low atomic weight and low absorption cross-sections for neutrons.

3) **Neutron reflector.** By the use of reflectors on the surface of reactors, leakage of neutrons can be very much reduced and the neutron flux in the interior can be increased. Materials of high scattering cross-section and low absorption cross-sections for neutrons.

4) **Cooling system.** The cooling system removes the heat evolved in the reactor core. This heat is evolved from the K.E. of the fission fragments when they are slowed down in the fissionable substance and moderator. The coolant or heat transfer agent (water, steam, He, Co_2, air and certain molten metals and alloys) is pumped through the reactor core. Then, through a heat exchanger, the coolant transfers heat to the secondary thermal system of the reactor.

5) **Control and safety system.** The control systems enable the chain reaction to be controlled and prevent it from spontaneously running away. This is accomplished by pushing control rods into the reactor core. These rods are of a material (boron or cadmium) having a large neutron-absorption cross-section. These rods absorb the neutrons and hence cut down the reactivity. By pushing in the rods, the operation of the reactor can be made to die down, by pulling the out to build up. The safety systems protect the space surrounding the reactor against intensive neutron flux and gamma rays existing in the reactor core. This is achieved by surrounding the reactor with massive walls of concrete and lead which would absorb neutrons and gamma rays.

Power reactor. The heat generated in a nuclear reactor is used for producing power in a nuclear power plant. A quantity of enriched uranium in the form of pure metal of solution of a soluble salt in water constitutes the center of heat energy source. A large quantity of heat is produced in the fission processes. The cadmium rods regulate the temperature, to a pre-determined value. If it is desired to bring down the temperature, the cadmium rods are pushed down further as to absorb more neutrons. If the temperature has to be raised,



the cadmium rods are pulled up a little. A fluid is circulated through the shielded reactor and heat exchanger. The hot fluid, while flowing through the heat exchanger, converts water into steam. The steam produced runs conventional turbines to produce electricity.

Breeder reactor. If a thermal reactor core with U^{235} fuel is surrounded by a blanket of a fertile material like U^{233}, U^{238} can be converted into fissile fuel. Reactors of this type are called fuel producing reactors. The reactions are as follow.

$$_{92}U^{238} + {}_0n^1 \longrightarrow {}_{92}U^{239} + \gamma$$

This is followed by $_{92}U^{239} \longrightarrow {}_{93}Np^{239} + \beta$

$_{93}Np^{239}$ is also radioactive. It emits a β-particle to form plutonium.

$$_{93}Np^{239} \longrightarrow {}_{91}Pu^{239} + \beta + \gamma$$

This process of producing one type of fissionable material (Pu^{239}) from a non-fissionable material (U^{238}) is called breeding and the reactor a breeder reactor.

Uses of nuclear reactors.

1) **Nuclear power.** Nuclear reactors are used in the production of electric energy.
2) **Production of radioisotopes.** Nuclear reactors are useful in producing a large number of radio-isotopes. To produce radio isotope, a suitable compound is drawn into the centre of the reactor core where the flux of neutrons may well more than 10^{16} /m^2/sec. Sodium-24 is manufactured in this way.
$${}_{11}Na^{23} + {}_{0}n^{1} \longrightarrow {}_{11}Na^{24}$$
3) **Scientific research.** Reactors produce a number of radioactive materials needed for research purposes. The reactors provide a huge source of neutrons. Using these neutron, several useful radioisotopes have been artificially produced and several nuclear reactions have been studied. We may also study the effect of neutrons on biological tissues. Reactors may also be used to study radiations damage.

Nuclear fusion

Nuclear fusion. In this process, two or more light nuclei combine together to form a single heavy nucleus. For example, when four hydrogen nuclei are fused together, a helium nucleus is formed. The mass of the single nucleus formed is always less than the sum of the masses of the individual light nuclei. The difference in mass is converted into energy according to Einstein's equation $E = mc^2$.

Example. Consider a single helium nucleus formed by two deuterium nuclei. Mass of ${}_1H^2$ = 2.01478 amu ; mass of ${}_2He^4$ = 4.00388 amu.

$${}_{1}H^{2} + {}_{1}H^{2} \longrightarrow {}_{2}He^{4}$$

The initial mass of two deuterium atoms = 2 × 2.01478 = 4.02956 amu.

Mass of helium atom = 4.00388 amu

∴ Decrease in mass = 4.02956 – 4.00388 = 0.02568 amu

∴ Energy released = 0.02568 × 931 MeV = 23.91 MeV.

Thus, the energy released in fusion is 23.91 MeV.

Source of stellar energy

The temperatures of the stars are very high and they radiate tremendous amount of energy. The sun is one of the innumerable stars. The sun radiates 3.8×10^{26} joules of energy each second. The origin of such a tremendous amount of energy is neither chemical nor gravitational. The fusion of protons is supposed to release the energy in the sun and in other stars. Bethe suggested the following carbon-nitrogen cycle as one of the most important nuclear reactions for release of energy by fusion.

Carbon-Nitrogen cycle. The cycle is as follow (Fig. 11.9).

$$_6C^{12} + {}_1H^1 \longrightarrow {}_7N^{13} + \gamma \qquad \ldots(1)$$

$$_7N^{13} \longrightarrow {}_6C^{13} + {}_1e^0 + \nu \qquad \ldots(2)$$

$$_6C^{13} + {}_1H^1 \longrightarrow {}_7N^{14} + \gamma \qquad \ldots(3)$$

$$_7N^{14} + {}_1H^1 \longrightarrow {}_8O^{15} + \gamma \qquad \ldots(4)$$

$$_8O^{15} \longrightarrow {}_7N^{15} + {}_1e^0 + \nu \qquad \ldots(5)$$

$$_7N^{15} + {}_1H^1 \longrightarrow {}_6C^{12} + {}_2He^4 \quad \ldots(6)$$

In this cycle, C acts like a catalyst.

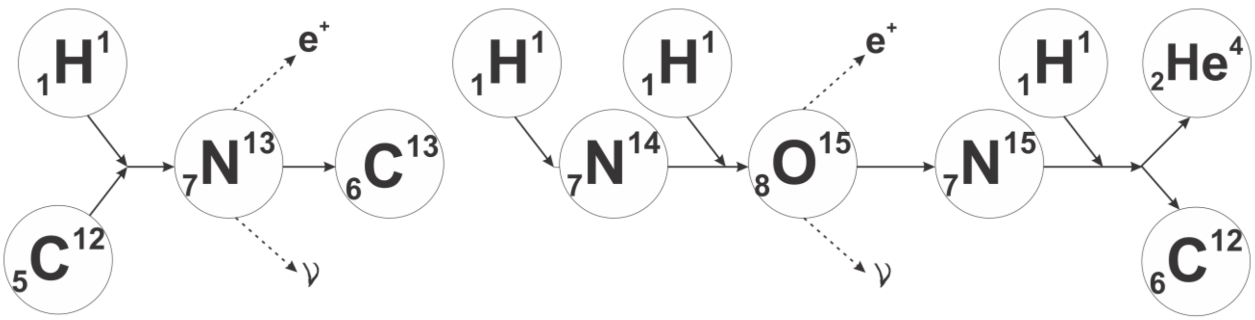


Fig.11.9

The reaction cycle is essentially the reaction

$$4{}_1H^1 \longrightarrow {}_2He^4 + 2{}_1e^0 + 2\nu + Q$$

The loss in mass is calculated as follow:

$$4{}_1H^1 = 4.031300;\ {}_2He^4 = 4.002603\ and\ 2{}_1e^0 = 0.001098.$$

∴ Loss in mass = 0.02756 amu.

∴ Energy released = 0.02756 × 931 = 25.7 MeV.

It is found that in one million years the sun loses about 10^{-7} of its mass by the above process. Taking mass of the sun as 2×10^{30} kg and its present age is 10^{10} years, it is estimated that the C-N cycle may keep going for another 30 billion years.

Proton-Proton cycle. Recent modification of the estimates of the central temperature of the sun now favor the Proton-Proton chain (where temp < 1.6×10^7 K). In the p-p chain, two protons first fuse to produce a deuterium nucleus which combines with another proton to yield

He^3. Two He^3 nuclei interact and form He^4 and two protons. These reactions can be represented by the equations.

$$_1H^1 + \ _1H^1 \longrightarrow (_2He^2) \longrightarrow \ _1H^2 + \ _1e^0 + \nu + 0.042\ MeV$$

$$_1H^2 + \ _1H^1 \longrightarrow (_2He^3) \longrightarrow \ _2He^3 + \gamma + 5.5\ MeV$$

$$_2He^3 + \ _2He^3 \longrightarrow (_4Be^6) \longrightarrow \ _2He^4 + \ _1H^1 + {}_1H^1 + 12.6\ MeV$$

$$4\,_1H^1 \longrightarrow \ _2He^4 + \ 2 + \ _1e^0 + 2\nu + 2\gamma + 26.7\ MeV$$

Thermonuclear reactions

The source of stellar energy is fusion. This suggests that a large amount of energy can be obtained by nuclear fusion. But it is not easy to fuse the light nuclei into a single nucleus. The main difficulty in the fusion of nuclei is the electric force of repulsion between the positively charged nuclei. Fusion is possible when the K.E of each of the nuclei is large enough to overcome the repulsion. Fusion reactions can take place only at very high temperatures (of the order of 10^7 to 10^9 K). only at these very high temperatures, the nuclei are able to overcome their mutual coulomb repulsion and enter the zone of nuclear attractive forces. Hence these reactions are called thermonuclear reactions.

A star is able to control thermonuclear fusion in its core because of its strong self-gravity. The thermonuclear reactions in the core of the sun cause high temperatures which generate strong outward pressures; these acts against the sun's own gravity, preventing it from contracting, and holding it in equilibrium. The equation of stellar structure, set up by A.S. Eddington, relates the gravitational force in the star to the progressive changes of pressure from its centre outwards, the magnitude of pressure to density and temperature, and the fall of temperature outwards to the flow of energy from the interior to the surface. From these equations, stable models of stars emerge, with central temperatures high enough to start and sustain thermonuclear fusion. The key role, of course, is played by controlling force of gravity. The large mass of an astronomical system makes gravity the most important factor in determining its behaviour.

Hydrogen bomb. Hydrogen bomb is a device which makes use of the principle of nuclear fusion. The very high temperature required for an uncontrolled thermonuclear reaction is obtained by the detonation of an atom bomb. In this weapon, hydrogen is the core. The fission bomb produces a very high temperature, at which thermonuclear reactions start resulting in the fusion of hydrogen nuclei to form helium. Greater energy per unit mass is obtained from a hydrogen bomb than from a nuclear fission bomb.

Controlled thermonuclear reactions. A large amount of energy is released in a fraction of a second in a hydrogen bomb. If the thermonuclear reaction could be controlled to take place more slowly, the energy released can be used for constructive purposes. We know that very high temperatures are needed to bring about a nuclear fusion process. The main problem is to produce such a high temperature and to find a container for the gas which can stand this temperature. At this temperature, the gas is highly ionized and is called plasma. One of the severe engineering problems is the design of a "container" in which a very hot plasma can be contained under high pressure to initiate a fusion reaction. Since almost any container would melt in the presence of plasma, attempts are being made to contain and controlled plasma trapped in a specially shaped magnetic field (Fig. 11.10).

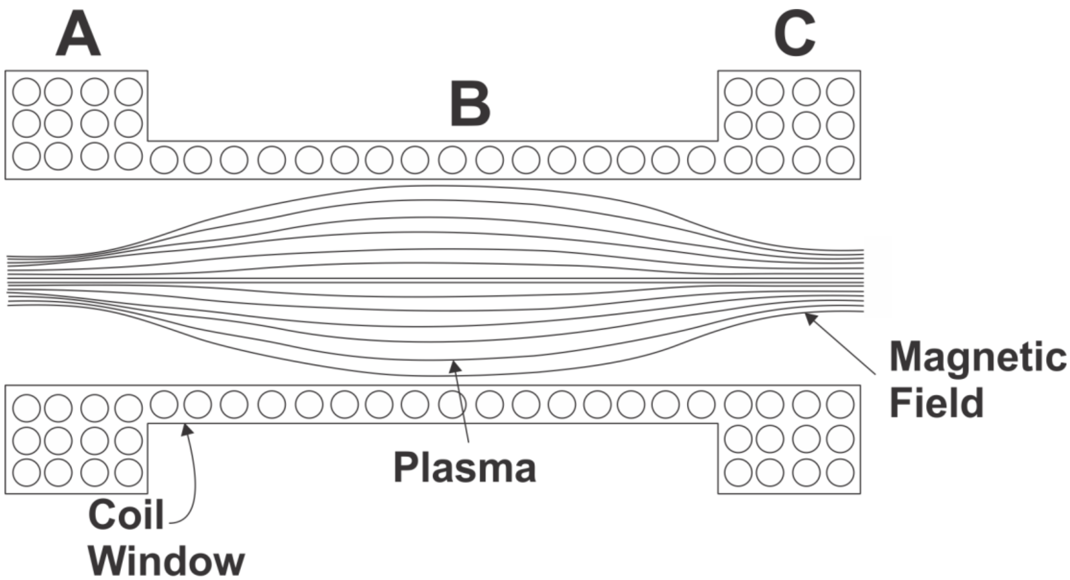


Fig. 11.10

By increasing the field and changing the shape of the field, it is hoped that the plasma in this "magnetic bottle" can be raised to the required temperature and pressure for fusion reactions.

Nuclear fusion as an energy source will be a boon to humanity because of the following reasons.

1) Hydrogen is available everywhere on this planet in various forms.
2) The lightness of the reactant nuclei makes the energy yield per unit mass of the reacting material much greater than that in nuclear fission process.
3) The dispersal of hydrogen over the whole surface of the planet and easy availability make the proposition very economically attractive.
4) A fusion reactor does not leave behind as in fission reactor radioactive waste, the disposal of which poses a tremendous problem.

Plasma

Plasma is the basic form of substances in the outer space (stars, nebulae "interstellar cloud of dust"). At the lowest temperatures, all substances exist in the solid state. When sufficient heat is added to a solid, it becomes liquid (fusion). At still higher temperatures, with more energy, the liquid in turn gets converted into gas. If more energy is added, at very high temperatures, the kinetic energy of the molecules increases. As a result, the atoms will come into violent collisions with each other and a fraction of the particles get dissociated into electrons and positive ions. The gas passes on to a state of matter, called the plasma state. Plasmas are sometimes regarded as constituting a fourth state of matter, since their physical properties differ so markedly from those of matter in the solid, liquid and neutral-gaseous states.

What is plasma. Plasma is a state of matter, characterized by a high, or even complete ionization of its particles. The degree of ionization (α) of a plasma is defined as the ratio of the concentration of ionized particles to the total concentration. Depending upon the degree of ionization α, a plasma is classified as:

1) Weakly ionized (α is a fraction of one percent)

2) Moderately ionized (α equals several percent)

3) Fully ionized (α is close to 100%)

Plasma is an assembly of ions, electrons, neutral atoms and molecules, in which the motion of the particles is dominated by electromagnetic interactions. The particles, atoms or molecules, may be singly, multiply ionized. Plasma contains approximately an equal quantity of positive and negative charges. Both the groups of charged particles need not necessarily be mobile. One group, say positively charged particles, may be stationary. Since a plasma consists of charged particles, it can conduct electricity. Plasma are good conductors of electricity and are strongly affected by magnetic fields.

Artificially, plasmas can be produced by heating gases to above 50,000^{o}C until all electrons are detached form the atoms of the gases. Plasmas can also be made by applying a voltage across a tube containing the gas or a vapour at low pressure.

Examples of plasmas in nature. It is estimated that more than 99% matter is the known universe is in the plasma state. The lightning, which accompanies thunderstorms, is a vivid example of plasma. The sun, hot stars and certain interstellar clouds are examples of fully ionized plasma, formed at very high temperatures (high temperature plasma). Solar flares, solar prominences and sun-spots are the other spectacular plasma phenomenon. The Van Allen belts around the earth are also plasmas. The Aurora Borealis provides an impressive plasma display in the earth's upper atmosphere.

Transuranic elements

Elements with their atomic numbers greater than that of uranium (z=92) are called transuranic elements. All these are man-made and radioactive. Some of these elements are fissionable and hence useful. The following is the list of transuranic elements.

Such transuranic elements may be produced in the laboratory by the bombardment of certain heavy nuclides with neutrons. We give below typical methods of production, the reactions involved and the radioactive decays of two of these nuclides.

Z = 93	94	95	96	97	98
Np	Pu	Am	Cm	Bk	Cf
Neptunium	Plutonium	Americium	Curium	Berkelium	Californium
Z = 99	100	101	102	103	104
Es	Fm	Md	No	Lw	Xv
Einsteinium	Fermium	Mendelevium	Nobelium	Lawrencium	Kurchatovium

1) Neptunium (Z = 93): When $_{92}U^{238}$ is bombarded with slow energy neutrons, neptunium is formed according to the reaction

$$_{92}U^{238} + {}_0n^1 \longrightarrow {}_{92}U^{239}$$

$$_{92}U^{239} \longrightarrow {}_{93}Np^{239} + {}_{-1}e^0$$

2) Plutonium (Z = 94): Neptunium ($_{93}Np^{239}$) is itself radioactive. It emits a β-particle and produces plutonium according to the reaction

$$_{93}Np^{239} \longrightarrow {}_{94}Pu^{239} + {}_{-1}e^0$$

Plutonium emits α-particles and decays into $_{92}U^{235}$ with a half-life of 240,000 years

$$_{94}Pu^{239} \longrightarrow {}_{92}U^{235} + {}_2He^4$$

Solved Problems

Problem 11.1. A U-235 nucleus is fissioned by a thermal neutron and two fission fragments and two neutrons are produced. Compute the fission energy released if the average binding energy per nucleon is 7.8 MeV in the fissioned U-235 nucleus and 8.6 MeV in the fission fragments.

Sol. Greater binding energy of the fission fragments indicates that there has been release of energy during fission of low binding energy nucleus U-235.

Fission energy released is = (234 × 8.6 – 236 ×7.8)

= 171.6 MeV

Problem 11.2. A reactor is developing energy at the rate of 3000 KW. How many atoms of U^{235} undergo fission per second? How many kilograms of U^{235} would be used in 1000 hours of operation assuming that on an average energy of 200 MeV is released per fission?

Sol. Rate of development of energy by the reactor = 300KW = 3×10^6 Js^{-1}

Energy released per fission = 200 MeV = $200 \times 1.6 \times 10^{-13}$ J = 32×10^{-12} J.

∴ Number of atoms undergoing fission per second = $\frac{3 \times 10^6}{32 \times 10^{-12}}$ = 9.4×10^{16}

Number of atoms undergoing fission in 1000 hours = $(9.4 \times 10^{16})(1000 \times 60 \times 60)$

= 3.384×10^{23}

According to Avogadro's hypothesis, 6.025×10^{26} atoms of U^{235} weight 235 kg.

∴ Weight of 3.384×10^{23} atoms of U^{235} = $\frac{235 \times (3.384 \times 10^{23})}{6.025 \times 10^{26}}$

= 0.1321 kg.

Problem 11.3. A deuterium reaction that occurs in experimental fusion reactor is H^2 (d,p) H^3 followed by H^3 (d,n) He^4. (a) Compute the energy release in each of these. (b) Compute the total energy release per gram of the deuteron used in the fusion. (C) Compute the percentage of the rest mass of deuteron released as energy. (d) Compare U^{235} fission with deuteron fission as a source of energy release. Given H^2 = 2.014740 amu, H^3 = 3.017005 amu, H^1 = 1.008145 amu, ${}_0n^1$ = 1.008986 amu and He^4 = 4.003179 amu. U^{235} = 235.1175 amu.

Sol. **(a) (i)** The fusion reaction H^2 (d,p) H^3 is

$$ {}_1H^2 + {}_1H^2 \longrightarrow {}_1H^3 + {}_1H + Q $$

Mass decrease in the reaction

= m = 2.014740 + 2.014740 - 3.017005 - 1.008145 = 0.004330 amu.

∴Energy released = 0.004330 × 931 MeV = 4.031 MeV.

(a) **(ii)** The reaction H^3 (d,n) He^4 is

$$_1H^3 + {}_1H^2 \longrightarrow {}_2He^4 + {}_0n^1 + Q$$

Δm = 3.017005 + 2.014740 - 4.003179 - 1.008986 = 0.01958

Energy released = 0.01958 × 931 = 18.23 MeV.

∴ Total energy released = 4.031 + 18.23 = 22.26 MeV.

(b) This total energy release is from the fusion of $3\,{}_1H^2$ nuclei.

∴ Energy release per H^2 = 22.26 = 7.42 MeV.

No. of nuclei in 1 gram of H^2 = $\frac{6.02 \times 10^{23}}{2.01474}$

∴ Total release of energy from 1 gram of H^2

$$= \frac{6.02 \times 10^{23}}{2.01474} \times 7.42 = 2.217 \times 10^{24} \text{ MeV}$$

(c) Energy equivalent of one H^2 nucleus

$$= 2.01474 \times 931 \text{ MeV}$$

Average release of energy per H^2 nucleus = 7.42 MeV

∴ The percentage of rest mass of deutron released as energy

$$= \frac{7.42}{2.01474 \times 931} \times 100 = 0.3956\%$$

(d) In U^{235} fission, 200 MeV is released per uranium nucleus.

∴ Percentage of mass energy release in U^{235} fission

$$= \frac{200}{235.1 \times 931} \times 100 = 0.09317\%$$

$$\therefore \frac{Energy\ release\ from\ H^2\ fusion}{Energy\ release\ from\ U^{235}\ fission} = \frac{0.3956}{0.09317} = 4.247$$

Problem 11.4 When $_{94}Pu^{239}$ is bombarded with α-particles two neutrons are liberated. What is the final product? What is the product if only one neutron is liberated?

Sol. The required equations are

$$_{94}Pu^{239} + {}_2He^4 \longrightarrow 2\,{}_0n^1 + {}_{96}Cm^{241}$$

$$_{94}Pu^{239} + {}_2He^4 \longrightarrow {}_0n^1 + {}_{96}Cm^{242}$$

Both curium isotopes are α-emitters with almost identical energy but with different half-lives.

EXERCISE

Q. 1. **Write an essay on nuclear fission and its important applications.**

Q. 2. **Distinguish between nuclear fission and fusion. Explain the principle of a nuclear reactor. Mention some of its uses.**

Q. 3. **Describe the construction and working of a nuclear reactor. When is, the reactor said to be critical?**

Q. 4. **Give an account of the discovery and properties of transuranic elements.**

Q. 5. **Why are neutrons moderated to thermal speeds in nuclear reactors?**

Q. 6. **What are the differences and similarities between nuclear fission and nuclear fusion?**

Radioactivity

Q. 3.1. (*a*) What is natural radioactivity? Explain what is radioactive disintegration (or decay). State the laws of radioactive decay and deduce them from first principles using probability concept.

(*b*) Define radioactive constant and half life period. Prove that the radioactive constant of a substance is the reciprocal of the time after which the numer of atoms of the substance falls to $\frac{1}{e}$ of its original value.

(*G.N.D.U.* 2004, 1997, 1995, *Merrut* 2003,2002, 2000, *M.D.U.* 2002, 1995, it. *P.U.* 2002, 1995, *Pbi.U.* 2001, 2000, *Calcutta.U.* 2002, *K.U.* 2002, 1992, *Lucknow.U.* 1996, *Magadh U.* 1996, *Patna.U.* 1991, *Cal.U.* 1991)

Ans. (*a*) **Natural radioactivity.** The phenomenon of spontaneous emission of highly penetrating radiation from heavy elements of atomic weight greater than about 206 occurring in nature is called *natural radioactivity.* The elements which exhibit this property are called *radioactive elements.*

Radioactive disintegration. Heavy elements like uranium, thorium, polonium, radium, etc., spontaneously disintegrate giving out α, β and γ radiations. In this way new atoms are formed. Uranium, on disintegration, gives rise to a new substance UX. It is found that when UX is just separated from the parent uranium, it is highly radioactive. *The* **activity** *of a substance is measured by the number of atoms that disintegrate per second.*

The activity of uranium is at first small as compared to UX, but with lapse of time, uranium regains its activity, whereas the activity of UX decreases. The decay and growth of activity with time is represented graphically in Fig. 3.1.

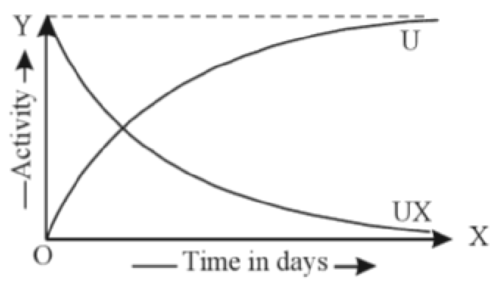


Fig. 3.1

It is seen that the two curves follow the exponential law.

For UX $\qquad I_t = I_0 e^{-\lambda t}$

and for U $\qquad I_t = I_0 (1 - e^{-\lambda t})$

where I_0 is the activity at a given instant, I_t the activity after a time t and λ is the radioactive constant.

Laws of radioactive decay (disintegration). The above facts can be explained on the following hypothesis due to Rutherford and Soddy.

(*i*) *Atoms of every radioactive substance are constantly breaking into fresh radioactive products with the emission of* α, β *and* γ*-rays.*

(*ii*) *The rate of breaking is not affected by external factors (temperature, pressure, chemical combinations, etc.) but is based upon probability concept and depends entirely on the law of chance i.e., the number of atoms breaking per second at any instant is proportional to the number present.*

If there are N atoms of any substance and a nuumber dN breaks in a time dt, then according to the probability hypothesis

Rate of breaking $R = \frac{dN}{dt} \propto N$

or $R = \frac{dN}{dt} = -\lambda N$ ...(*i*)

where λ is the *radioactive constant or* (*disintegration constant or decay constant*)

(*b*) **Radioactive constant.** *It is defined as the ratio of the amount of the substance which disintegrates in a unit time to the amount of the substance present.*

$\therefore$ Radioactive constant $\lambda = \frac{-\frac{dN}{dt}}{N}$

Relation (*i*) can also be put in the form

$$\lambda = \frac{R}{N} \qquad \text{(numerically)}$$

Thus, we see that the number of atoms decaying at any time is always proportional to the number of atoms present.

λ is also known as *decay constant or disintegration* constant.

Exponential decay. The differential equation (*i*) can be put in the form

$$\frac{dN}{N} = -\lambda dt$$

Intergrating, we have

$$\log_e N = -\lambda t + C \qquad ...(ii)$$

where C is the constant of intergration.

When $t = 0, N = N_0$

$\therefore$ $\log_e N_0 = C$

Substituting in (*ii*), we get

$$\log_e \frac{N}{N_0} = -\lambda t$$

or $N = N_0 e^{-\lambda t}$ ...(*iii*)

$\therefore$ $\frac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$ ...(*iii*)(*a*)

Since dN/dt measures the activity of a radioactive substance and is proportional to the number of atoms present, equation (*iii*) explains the decay curve in Fig. 3.1. It will be observed that the equations $I_t = I_0 e^{-\lambda t}$ and $N = N_0 e^{-\lambda t}$ are exactly similar. Thus, it shows that a radioactive element decays exponentially with time.

Exponential growth. From relation (*iii*) we also find that out of a number of atoms N_0 at time $t = 0$ the number left after a time t is N. It means that $N_0 - N$ atoms of the parent substance after disintegration are converted into the daughter (or product) substance in time t. If we assume that the daughter is not radioactive, then the rate of growth of daughter

$$N_0 - N = N_0 - N_0 e^{-\lambda t} = N_0 (1 - e^{-\lambda t}) \qquad ...(iv)$$

This equation is similar to $I_t = I_0 (1 - e^{-\lambda t})$. Thus, it shows that the daughter product of a radioactive element grows exponentially with time.

For half life period see **Q. 3.3 (*b*)**.

From relation (*iii*), we have $N = N_0 e^{-\lambda t}$

Therefore, after a time $t = \frac{1}{\lambda}$, the number of atoms of the radioactive substance left behind is given by

$$N = N_0 e^{-\lambda \times \frac{1}{\lambda}} = \frac{N_0}{e}$$

$$\therefore \qquad \frac{N}{N_0} = \frac{1}{e}$$

Hence the radioactive constant is also defined as the reciprocal of the time during which the number of atoms of a radioactive substance falls to $\frac{1}{e}$ *of its original value.*

Q. 3.2. Radioactivity is a random phenomenon. Elaborate it. (*Luck.U.* 1992)

Ans. It has been proved that radioactive deacy follows the exponential law. This fact implies that the phenomenon is statistical in nature. Every nucleus in a sample of radioactive substance has a certain probability of decay but *there is no way of knowing in advance which nuclei will actually decay in a particular span of time.*

Thus, in other words it means that radioactivity is a random phenomenon.

Q. 3.3. (*a*) Define mean (or average) life of a radioactive nuclide. Derive a relation between mean life and radioactive constant.

(*b*) Define half life of a radioactive nuclide. Derive a relation between half life and radioactive constant.

(*c*) What is the difference between half life and mean life in radioactivity?

Ans. (*a*) Average (or mean) life of a radioactive substance. The atoms of a radioactive substance are constantly disintegrating and thus the life of every atom is different.

When a radioactive substance is separated from its disintegration product, the atoms which disintegrate earlier have a very short life and others which disintegrate at the end have a long life. The average or mean life of a radioactive substance is equal to

$$\frac{\text{The sum of the lives of all the atoms}}{\text{Total number of atoms}}$$

The average (or mean) life of a radioactive substance is defined as the ratio of the total life time of all the radioactive atoms to the total number of such atoms in it.

Relation between average life T_a and radioactive constant λ. Suppose dN is the number of atoms disintegrating in a time dt seconds after the separation of the substance, when the actual number of atoms present is N, then

$$\frac{dN}{dt} = -\lambda N$$

or

$$-dN = \lambda N\, dt$$

$$= \lambda N_0 e^{-\lambda t}\, dt$$

where N_0 is the number of atoms in the beginning of time t.

Each of these $-dN$ atoms disintegrated between the time t and $t + dt$, *i.e.*, these atoms have a life of t seconds.

$\therefore$ Total life of $-dN$ atoms $= -t\,dN$.

Since the possible life of any one of the total number of atoms varies from 0 to ∞ the total life of all N_0 atoms is given by

$$\int_0^{N_0} -t\,dN$$

$\therefore$ Average life

$$T_a = \frac{1}{N_0}\int_0^{N_0} -t\,dN$$

$$= \frac{1}{N_0}\int_0^{\infty} \lambda N_0 e^{-\lambda t}\,t\,dt$$

$$= \lambda\int_0^{\infty} e^{-\lambda t}\,t\,dt$$

$$= \lambda\left[\frac{e^{-\lambda t}}{-\lambda}t - \int\frac{e^{-\lambda t}}{-\lambda}dt\right]_0^{\infty}$$

$$= \lambda\left[\frac{e^{-\lambda t}}{-\lambda}t - \frac{e^{-\lambda t}}{\lambda^2}\right]_0^{\infty}$$

$$= -\frac{1}{\lambda}[(\lambda t + 1)\,e^{-\lambda t}]_0^{\infty} = \frac{1}{\lambda}$$

Thus *the mean life of a radioactive substance is the reciprocal of the radioactive constant i..e.*

$$T_a = \frac{1}{\lambda} \qquad ...(i)$$

or

$$\lambda = \frac{R}{N} = \frac{1}{T_a}$$

(*b*) Half -life period. *The half-life period of a radioactive substance is the time in which half of the radioactive substance will disintegrate.*

Relation between half life period and radioactive constant. To determine the half life period we have to find the time T in which the number of atoms N left behind becomes equal to $N_0/2$ where N_0 is the number of atoms in the beginning.

Now $N = N_0 e^{-\lambda t}$

$\therefore$ $\frac{N_0}{2} = N_0 e^{-\lambda T}$

or $\frac{1}{2} = e^{-\lambda T}$

or $2 = e^{+\lambda T}$

or $\lambda T = \log_e 2$

Hence

$$T = \frac{\log_e 2}{\lambda} = \frac{0.6931}{\lambda} \qquad ...(iii)$$

(*c*) Difference between half life and mean life. The half life and mean life of a radioactive substance are two different quantities.

Half life $T = \frac{0.6931}{\lambda}$ ...(Eq. *iii*)

Mean life $T_a = \frac{1}{\lambda}$ ...(Eq. *i*)

ematics

$$\therefore \quad T = 0.6931\, T_a$$

i.e. Half life = 0.6931 × Mean life

Also $$T_a = \frac{T}{0.6931} = 1.44\, T$$

i.e. Mean life = 1.44 × Half life.

Q. 3.4. (*a*) The half life of $_{92}U^{238}$ is 4.51×10^9 years. What percentage of $_{92}U^{238}$ that existed 10^{10} years ago still survives. (*K.U.* 1991)

(*b*) One mg of radioactive material with half life of 1600 years is kept for 2000 years. Calculate the mass which would have decayed by this time. (*P.U.* 2002)

Ans. (*a*) Radioactive constant of $_{92}U^{238} = \lambda = \frac{0.6931}{\text{Half life}} = \frac{0.6931}{4.51 \times 10^9}\ \text{yr}^{-1}$

If N_0 is the number of atoms of $_{92}U^{238}$ that existed 10^{10} years ago and N is the number now present, then

$$N = N_0 e^{-\lambda t} \text{ where } t = 10^{10} \text{ years.}$$

or $$\frac{N_0}{N} = e^{+\lambda t}$$

or $$\log_e \frac{N_0}{N} = \lambda t$$

or $$2.3026 \log_{10} \frac{N_0}{N} = \lambda t = \frac{0.6931 \times 10^{10}}{4.51 \times 10^9}$$

or $$\log_{10} \frac{N_0}{N} = \frac{0.6931 \times 10}{2.3026 \times 4.51} = 0.6673$$

$$\therefore \quad \frac{N_0}{N} = \text{Antilog } 0.6673 = 4.648$$

or $$\frac{N}{N_0} = \frac{1}{4.648} = 0.215$$

∴ % age of $_{92}U^{235}$ now present = 0.215 × 100 = 21.5%.

(*b*) Half life of radioactive material T = 1600 years.

∴ Radioactive constant $\lambda = \frac{0.6931}{T} = \frac{0.6931}{1600}\ yr^{-1}$

Let N be the number of atoms of the radioactive material left behind after 2000 years and N_0 the number in one mg in the beginning

$$\therefore \quad \frac{N}{N_0} = e^{-\lambda t} \text{ or } \log_e \frac{N_0}{N} = \lambda t$$

$$= \frac{0.6931}{1600} \times 2000 = 0.866$$

$$\therefore \quad \frac{N_0}{N} = 2.378$$

or $$N = \frac{N_0}{2.378} = \frac{1}{2.378} = 0.42 \text{ mg}$$

$\therefore$ The mass which has decayed

$$= 1 - 0.42 = 0.58 \text{ mg}$$

Q. 3.5. The half-value period of radium is 1590 years. In how many years will one gram of pure element

(*i*) lose one centigram and

(*ii*) be reduced to one centigram? (*P.U.* 1990)

Ans. Half life period of radium $T = 1590$ years

$$\text{Radioactive constant } \lambda = \frac{0.6931}{T} = \frac{0.6931}{1590} \text{ yr}^{-1}.$$

(*i*) Let t be the time in which one gram of radium loses one centigram (0.01 gm.).

$\therefore$ Radium left behind

$$= 1 - 0.01 = 0.99 \text{ gm.}$$

Now $$N = N_0 e^{-\lambda t}$$

or $$\log_e N = \log_e N_0 - \lambda t$$

or $$\lambda t = \log_e \left(\frac{N_0}{N}\right)$$

$\therefore$ $$t = \frac{1}{\lambda} \log_e \left(\frac{1}{0.99}\right)$$

$$= \frac{1590}{0.6931} \log_e \left(\frac{100}{99}\right) = \frac{1590 \times 2.3026}{0.6931} \log_{10} \left(\frac{1}{0.99}\right)$$

$$= \frac{1590 \times 2.3026 \times 0.0044}{0.6931} = 23.25 \text{ years.}$$

(*ii*) When it is reduced to one centigram

$$N = 0.01 \text{ gram}$$

$\therefore$ $$\lambda t = \log_e \frac{1}{0.01}$$

or $$t = \frac{1590 \times 2.3026}{0.6931} \times \log_{10} (100)$$

$$= \frac{1590 \times 2.3026 \times 2}{0.6931}$$

$$= 10560 \text{ years.}$$

Q. 3.6. The half-life of $_{11}Na^{24}$ is 15 hrs. How long does it take for 93.75 per cent of a sample of this isotope to decay? (*P.U.* 1997; *G.N.D.U.* 1997; *Pbi. U.* 1997)

Q.3.7. Calculate the half life time and mean life time of the radioactive substance whose decay constant is 4.28×10^{-4} per year.

Q.3.8. The half life of a radioactive substance is 5 hrs. what will be its third life time?

Q. 3.9. Half life of a radioactive element is 4 years. After what time the element present in a specimen will reduce to $\frac{1}{64}$ of its original mass.

Q. 3.10. The half life of Radon gas is 3.8 days. Is it true that it will vanish in 8 days? Discuss your answer.

Q. 3.11. The activity of certain radio nuclide decreases to 15% of its original value in 10 days. Find its half life? (*G.N.D.U.* 1997)

Ans. Let N_0 be the original number of nuclei and N left behind after 10 days. If λ is the radioactive constant, then

$$\frac{N}{N_0} = e^{-\lambda t} \text{ or } \frac{15}{100} = e^{-\lambda 10}$$

or $$\log_e \frac{100}{15} = 10\lambda$$

$\therefore$ $$\lambda = \frac{1}{10}\log_e \frac{100}{15} = \frac{1}{10} \times 2.3026 \log_{10} \frac{100}{15} = 0.1897$$

$\therefore$ $$\text{Half life} = \frac{0.6931}{\lambda} = \frac{0.6931}{0.1897} = 3.65 \text{ days.}$$

Q. 3.12. The half life of a radioactive substance is 15 years. Calculate the period in which 2.5% of the initial quantity will be left over. (*Luck. U.* 1996)

Ans. Let N_0 be the initial number of nuclei and N left over after a time t, where $N = 2.5\%$.

$\therefore$ $$\frac{N}{N_0} = \frac{2.5}{100} = \frac{1}{40}$$

Half life $$T = 15 \text{ year}$$

$\therefore$ Radioactive constant $$\lambda = \frac{0.6931}{\lambda} = \frac{0.6931}{15} = 0.0462 \text{ year}^{-1}$$

Hence $$\frac{N}{N_0} = e^{-\lambda t} \text{ or } \frac{1}{40} = e^{-0.0462}$$

or $$\log_e \frac{1}{40} = -0.0462\, t \text{ or } \log_e 40 = 0.0462\, t$$

$\therefore$ $$t = \frac{\log_e 40}{0.0462} = \frac{2.3026 \log_{10} 40}{0.0462} = \frac{2.3026 \times 1.60206}{0.0462}$$
$$= 79.846 \text{ years.}$$

Q. 3.13. A certain radioactive element disintergrates for an interval of time equal to its mean life. What fraction of the element remains? What fraction has disintegrated?

Ans. Mean life $T_m = \frac{1}{\lambda}$

If a radioactive elements orginally having N_0 nuclei disintegrates for a time t so that number of nuclei left is N, then

$$\frac{N}{N_0} = e^{-\lambda t}$$

In this case $$t = T_m = \frac{1}{\lambda}$$

$$\therefore \qquad \frac{N}{N_0} = e^{-\lambda \times \frac{1}{\lambda}} = e^{-1} = \frac{1}{e}$$

$\therefore$ Fraction of the element that remains = $\frac{1}{e}$

Hence fraction that has disintegrated = $1 - \frac{1}{e} = \frac{e-1}{e.}$.

Q. 3.14. A certain radioactive substance has a half life of 30 days. What is its disingration constant? *(P.U.* 2001)

Ans. Half life $T = \frac{0.6931}{\lambda}$ where λ is the disintegration (or decay) constant.

$$\therefore \qquad \lambda = \frac{0.6931}{T} = \frac{0.6931}{30} = 0.0231 \text{ day}^{-1}$$

Exercise 1. *Potassium 40 has a half life of 4 × 10⁸ years. Calculate the decay constant.* *(Merrut. U.* 2002)

Hint. $$\lambda = \frac{0.6931}{T} = \frac{0.6931}{4 \times 10^8 \times (365 \times 24 \times 60 \times 60)} \text{ sec}^{-1}$$

$$= 5.5 \times 10^{-17} \text{ sec}^{-1}$$

Exercise 2. *One gm of $_{90}Th^{232}$ emits 4500 α-particles per sec. Calculate the half life of the same.* *(Bang. U.* 2001)

Hint. Number of atoms in one gm of $_{90}\text{Th}^{232}$, $N_0 = \frac{6.023 \times 10^{23}}{232} = 2.6 \times 10^{21}$

Number of α-particles emitted per second = Number of atoms decaying per sec = $-\frac{dN}{dt} = 4500$

Now $\frac{dN}{dt} = -\lambda N \quad \therefore \ \lambda = \frac{dN}{dt} / N = \frac{4500}{2.6 \times 10^{21}} = 1.73 \times 10^{-18} \text{ sec}^{-1}$

Half life $$T = \frac{0.6931}{\lambda} = \frac{0.6931}{1.73 \times 10^{-18}} = 0.3994 \times 10^{18}$$

$$= \frac{0.3994 \times 10^{18}}{365 \times 24 \times 60 \times 60} = 1.266 \times 10^{10} \text{ years.}$$

Q. 3.15. Define activity as applied to the process of radioactive decay. Define main units of measuring intensity of radioactivity.

(*P.bi. U.* 1999; *G.N.D.U.* 1995, 1994; *Merrut U.* 2003; *H.P.U.* 2000, 1997; *P.U.* 2002; *K.U.* 2001; *Bang.U.* 2001)

Ans. Activity. In the process of radioactive decay, *the activity of radioactive sample is measured by the number of atoms that disintegrate per second.*

If dN is the number of atoms disintegrating in time dt second, then

Activity $$R = -\frac{dN}{dt}$$

Also $$\frac{dN}{dt} = -\lambda N = -\lambda\ N_0\, e^{-\lambda t}$$

$\therefore$ $$R = \lambda N = \lambda N_0\, e^{-\lambda t}$$

Unit of radioactivity. The S.I. units of radioactivity is called **Bacquerel.**

1 Bacquerel = 1 Bq = 1 disintegration per second

= 1 event per second

For very high disintegration activities met with in practice bigger units like $MBq = 10^6\ Bq$ and $G\ Bq = 10^9\ Bq$ are used.

The common unit for measuring activity is the **Curie.** *It is defined as 3.70×10^{10} disintegrations per second.* Its smaller units are millicurie, which is equal to 10^{-3} curie = 3.70×10^7 disintegrations per second and **micro-curie** = 10^{-6} curie.

1 curie = 1 Ci = 3.70×10^{10} events per second

= 37 $G\ Bq$

A unit **Rutherford** *which is equal to 10^6 disintegrations per second is also used.*

1 Rutherford (rd) = 10^6 disintegrations per second.

The smaller units are **milli-Rutherford** equal to 10^3 disintegrations per second and micro-Rutherford equal to one disintegration per second.

1 micro Rutherford = 1 Bq.

Q. 3.16. Write a short note on radioactive series.

Ans. Radioactive series. Most of the radioactive nuclides found in nature are members of four radioactive series. The first member of the series is called parent, the intermediate members are called *daughters* and the final stable member is called the *end-product.* These series are

1. Uranium series 2. Actinium series 3. Thorium series 4. Neptunium series.

Let us see why there are exactly four series. The decay by α-disintegration reduces the mass number of a nucleus by 4. Thus the nuclides whose mass numbers are all given by $A = 4\,n$ where n is an integer can decay into one another in descending order of mass number. All such radioactive nuclides whose mass number obeys the relation $A = 4\,n$ are said to be members of ($4n$) series.

Radioactive nuclides whose mass numbers obey the relation $A = 4n + 1$ belong to $(4n + 1)$ series. Similarly radioactive nuclides whose mass numbers satisfy the relation $A = 4n + 2$ and $A = 4n + 3$ belong to $(4n + 2)$ and $(4n + 3)$ series respectively. The four important radio-active series, their parent nuclides, half life of the parent and the final end product is given below.

Mass number	*Series*	*parent*	*Half life in years*	*Stable end-product*
$4n$	Thorium	${}_{90}Th^{232}$	1.39×10^{10}	${}_{82}Pb^{208}$
$4n + 1$	Neptunium	${}_{93}Np^{237}$	2.25×10^{6}	${}_{83}Bi^{209}$
$4n + 2$	Uranium	${}_{92}U^{238}$	4.51×10^{9}	${}_{82}Pb^{206}$
$4n + 3$	Actinium	${}_{92}U^{235}$	7.07×10^{8}	${}_{82}Pb^{207}$

Q. 3.17. What is radioactive decay series?

(*ii*) β-rays consist of electrons moving with a velocity of the order of 1% to 99% of the velocity of light. As these rays consist of electrons they have a negative charge equal to 1.6×10^{-19} coulomb and the same mass as that of electron.

(*iii*) γ-rays are electromagnetic waves of very short wavelength ranging from 0.005 to 0.5 Å.

Properties of α-rays. (*i*) The α-rays are shot out from the radioactive material with large velocities ranging from 1.4×10^7 to 1.7×10^7 metres per second.

The velocity of α-rays depends upon the radioactive substance from which they are ejected and from a given substance it is always the same.

(*ii*) *They produce intense ionisation* in the gas through which they pass. Their ionising power is 100 times greater than that of β-rays and 10,000 times greater than that of γ-rays. A thickness of 0.0005 cm. of aluminium foil reduces the ionising power to half. The tracks of α-particles observed by means of a *Wilson cloud chamber* are continuous, thick, straight lines with a slight bend in the end in certain cases.

Q. 3.39. What radiation do we get from radioactive substances. Give an account of their nature and properties. (*G.N.D.U.* 2004; *Calicut. U.* 2002; *P.U.* 1997; *Luck. U.* 1993)

Ans. The radiations emitted by radioactive substances are of three types (*i*) α-rays, (*ii*) β-rays and (*iii*) γ-rays.

Nature. (*i*) Alpha rays consist of nuclei of helium atoms ${}_2He^4$ moving with velocities of the order of 10^7 ms^{-1}. They have a positive charge twice in magnitude of the charge on an electron and mass equal to 4 atomic mass units.

(*iii*) They affect a photographic plate. The effect is, of course, very feeble.

(*iv*) *They produce fluorescence* in substance like zinc sulphide or barium platinocyanide. On observing the fluorescence through a low power microscope, *i.e.*, *spinthariscope* it is found to consist of successive *scintillations*, produced by the impact of an individual α-particle. The impact of a single α particle can thus be observed and the number of such particles given out per second by a radioactive substance can be found out. The fact that these scintillations are due to α-particles and not due to β or γ-rays can be proved by interposing a thin sheet of mica between the radium salt and the fluorescent screen. Since α-rays are absorbed by mica sheet the scintillations disappear.

(*v*) *Range of* α *particles*. Scintillations from α-particle cease abruptly after it has traversed a certain distance though matter. *The distance that an α-particle can travel in air, at atmospheric pressure is called its range in air*. It varies from 2.70 cms for α-particles given by uranium to 8.62 cms. for those of thorium C. The range of an α-particle depends upon

(*a*) The radioactive substance from which the rays are given out.

(*b*) The nature of the medium through which the rays travel.

(*c*) Velocity of emission. The range is proportional to V^3.

The number of α-particle in a beam remains constant up to the end of the range, but their energy goes on decreasing. Ultimately the α-particles do not have sufficient energy to produce either scintillations or ionisation.

The ionisation first increases slightly, reaches a maximum and then suddenly drops almost to zero value. The distance at which the ionisation drops almost to zero gives the range of α-particles.

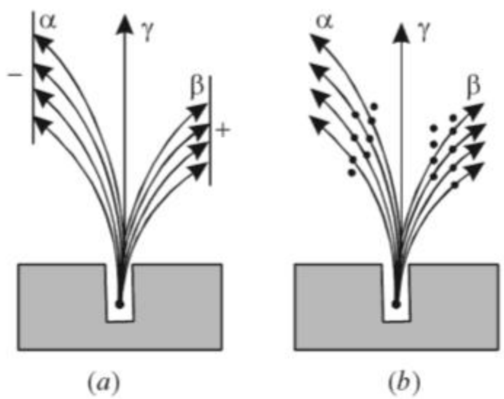


Fig. 3.2.

(*vi*) The α-rays are scattered when they pass through thin sheets of mica, gold foil, etc. The divergence of α-particle from its straight line path is 2 to 3 degrees. Geiger and Marsden found that

A few particles, sometimes, were deflected by more than 90°. this was explained by Rutherford to be duet to the repulsion between the αi-particles and the nucleus of the atom scattering it.

(vii) The rays are deflected by electric and magnetic fields showing that they are charged particles. If a narrow beam of radioactive radiations proceeding from a deep cavity in a block of lead is subjected to an electric field between parallel plates, the α-particles are deflected towards the negative plate as it fig. 3.2. This shows that α-rays consists of positively charged particles.

If a magnetic field is applied perpendicular to the plane of the paper and direction from top to bottom the α-rays are deflected towards the left as shown in figure 3.2 (b). Application of Fleming's left hand rule indicated that α-rays consists of positively charged particles. The value of e/m is determined on the same lines as for positive rays and indicated that α-particles are nuclei of helium atoms.

(viii) They produce a heating effect; A quantity of radium always maintains itself at a temperature higher than that of the surroundings. The evolution of heat is due to the stoppage of α, β and γ rays by the radioactive substance.

(ix) When exposed to α-rays the body suffers incurable burns.

Properties of $\boldsymbol{\beta}$-rays. (*i*) β-rays are shot our from radioactive substances with very high velocities ranging from 1% to 99% of the velocity of light. The velocity of all the β-particles given out by a substance is not the same.

(*ii*) They produce ionization in air but the number of ions produced is hardly $\frac{1}{100}$th of those produced by α-rays. Although their velocity is very large they posses a comparatively small mass ($\frac{1}{700}$th of an α-particle) and hence have a small kinetic energy. As β-particles are slowed down by collision with the atoms of the gas and change their path, their tracks in a Wilson cloud chamber are scattered and not continuous as are for the α-particles.

(iii) They affect a photographic plate and their effect is greater than those of α-rays.

(iv) They produce fluorescence in barium platinocyanide, calcium tungstate, willemite, etc.

(v) They can penetrate through large thickness of matter, e.g., they can easily pass through 1 cm thickness of aluminum sheet.

(vi) They are more readily scattered when they pass through matter, because their mass is very small as compared to the mass of the atomic nuclei.

(vii) They are affected by electric and magnetic fields. Their direction of deflection indicates that they are negatively charged particles. The e/m value has been found on the same lines as for cathode rays and indicates that β-rays are fast-moving electrons.

Properties of $\boldsymbol{\gamma}$-rays. (*i*) They posses the same velocity as that of light i.e., 3×10^8 metres/sec.

(ii) They ionize the gas through which they pass but the ionization produced is very small.

(iii) They affect a photographic plate and their effect is greater than those for β-rays.

(iv) They produce fluorescence in barium plationocyanido, etc.

(v) They are more penetrating than even β-rays and can pass easily through 30 cms, thickness of iron.

(vi) They are not affected by electric and magnetic fields. This property shows that they are electromagnetic waves of very short wavelengths.

(vii) They are diffracted by crystals in the same way as X-rays.

Q. 3.19. What are α-particles? How will you show experimentally that α-particles is an ionized helium atoms?

Ans. α-particles. The application of electric and magnetic fields to a beam of α-rays given out by a radioactive material shows that they consist of *positively* charged particles. The value of e/m and velocity for α-particles has been determined on the line of positive rays and it has been found that α-rays from all substances are of a *similar character*.

The value of the charge on an α-particle in 3.2×10^{-19} Coulomb. This shows that the charge is nearly double that on an electron. The value of e/m is 4.8×10^{7} C/kg which gives the mass of an α-particle to be 6.60×10^{-27} kg. The mass of a helium atoms is also 6.60×10^{-27} kg. Hence it is simplest to assume that α-particle is an atom of helium which has lost two electrons, that is, *it is the nucleus of the helium atom*.

Experimental proof. Rutherford has given a confirmatory proof of the *identity of α-particles with helium nuclei*. A thin-walled glass tube A is sealed into an outer tube B which is highly evacuated.

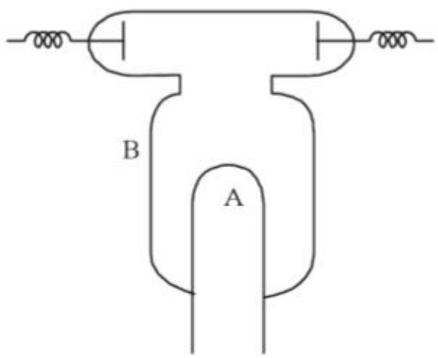


Fig. 3.3

The tube A is so thin that α-particles can pass through it, but it can withstand the atmospheric pressure. Two electrodes are sealed into the outer tube as shown in Fig. 3.3. A radioactive substance is placed in A. α-particle after passing through thin walls combine with electrons from the residual gas in B and are converted into helium atoms. On passing a discharge between the electrodes, characteristic spectrum of helium is observed.